I. Introduction and Dynamics Fundamentals
   A. Welcome
   B. What we will do today
      1. Introduction: Syllabus
      2. Talk about design project
      3. Review of dynamics fundamentals you should already know
   C. Go over syllabus

D. Design Project
   1. To be Determined

E. Assignment
   1. Problem 10-1
      2. Also, if the mallet is spun at 200 rpm about the tip of the handle, how much force is exerted by the handle to keep the mallet head on?

F. Dynamics Fundamentals
   10.0
      a. Kinematics

      b. Kinetics, or dynamic force analysis

      10.1 Newton's Laws of Motion

         a. Who was Newton?

         b. Laws
            i.

            ii.
iii.

c. Forward dynamics problem

known to be found

d. Inverse dynamics (kinetostatics) problem

known to be found

e. In the design process, the two cannot be completely separated

Possible design process for rotating machines

10.2 Dynamic Models

a. Simplified models used to describe motion of real parts

b. Three rules for such models

i. Equal _________

ii. Equal _____________________

iii. Same ___________________________

10.3 Mass

a. Not weight--what's the difference?

b. What are the units?

c. \( f = m a \)

d. \( m = \rho V \)

10.4 Center of Gravity (Center of Mass)

a. First moment of mass about an axis is equal to ___________ at the center of mass

b. Formula for the center of mass
c. Center of mass of a composite body

10.5 Mass Moment of Inertia

a. Rotational version of Newton's second law

b. Formula

c. Tabulated values for mass moment of inertia of common shapes:
   Appendix B, pp. 677-678

d. Kinetic energy
   \[ KE = \]
   \[ KE = \]

10.6 Parallel Axis Theorem

a. \[ I_{zz} = I_{gg} + m d^2 \]

b. Example
II. Fundamentals of Dynamics, Part II

A. Assignment. Empirically find the hoe’s MOI and center of rotation corresponding to a center of percussion at the edge of the hoe blade. You may work together to measure hoe properties, but each student must complete the problem himself. Above and beyond: design an attachment to relocate the center of rotation to a more useful position.

B. 10.7 Radius of Gyration

a. Definition
The radius at which the entire ______ of the body could be concentrated such that the resulting model will [sic] have the same ___________________ as the original body.

b. Formula

c. Depends upon the axis about which body is being rotated.

10.8 Center of percussion

a. Definition
_________________________ is a point on a body which, when struck with a force, will have associated with it another point called the _________________ at which there will be a zero reaction force.

b. Hockey stick example

\[ x = \]
\[ l_p = \]
\[ k = \]
\[ x = \]

The acceleration at R due to rotation about the CG is equal and opposite to the ______________ at R due to ______________.
c. Interpretation of formula

i. Negative sign?

ii. x small?

iii. x large?

iv. Symmetry?

v. What if the radius of gyration is small (mass is highly concentrated)?

vi. What if the radius of gyration is large (mass is highly distributed)?

10.11 Solution Methods
a. Superposition
   i. Description

   ii. Quick example

b. Matrix
   i. Description

   ii. Quick example
10.12 The Principle of D'Alemert
   a. Quasi-static formula

   b. System looks like a statics problem
      i. Used in graphical solutions to achieve equilibrium
      ii. Useful intuitive concept, especially for balancing
      iii. Example: free-body diagram and kinematic diagram combined into a quasi-static free-body diagram

   c. Centrifugal force

d. Centripetal acceleration

10.13 Energy methods
   a. Convert to power

   b. Equate power dissipation with power input

   c. Collect on left--virtual work equation (really a power equation)

   d. Only unknown is the _______________. This is the only thing you can solve for unless a special method is used. Good method if this is all you want, or you are solving the forward dynamics problem.

   e. Can be given in terms of generalized displacements, velocities--Kane's method

   f. One step further--LaGrange's equation
III. Newtonian Solution Method; 3-bar Linkage; Intro to Project

A. Assignment: Fill in the details for Example 11-2 in the text.
   1. Draw the free-body diagrams.
   2. Show your work in finding the magnitude and direction for each of the position vectors.
   3. Show your work in obtaining the magnitude and direction \( \mathbf{a}_{G2} \).
   4. What section in the text shows how to get \( \mathbf{a}_{G3} \)?
   5. Write out by hand each of the equilibrium equations which go into the matrix system.
   6. Solve the matrix equation using TK Solver, MATRIX, MATLAB, or other software. Provide hard copy of the set-up and the solution.

B.

11.1 Newtonian Solution Method
   a. Vector form of Newton's laws
   b. Component form of Newton's laws

11.2 Single Link in Pure Rotation
   a. Kinematic diagram
   b. Free-body diagram
   c. Force and torque equilibrium equations
      i. Equations
ii. Unknowns

d. Matrix formulation

e. Example

11.3 Force Analysis of a Three-bar Crank-Slider

a. Kinematic diagram

i. How are they getting the accelerations?

b. Free-body diagrams
c. Equations of equilibrium

d. Matrices

e. Example

B. Introduction to Design Project
Lecture 4

IV. Force Analysis of Fourbar Linkages
   A. Assignment (Lectures 4 and 5)
      Create a TK or Matlab program to solve kinematics and dynamic forces of a 4-bar mechanism. Solve 11-5b. Then create list variables $\theta$, $F_{px}$, and $F_{py}$ to solve the forces in the rotating mechanism in all positions.

   B. 11.4 Force Analysis of a Fourbar Linkage
      a. Previous kinematic analysis

      b. Three moving links

      c. _______ equations, _________ unknowns

      d. Free-body diagram
e. Unknowns: pin forces and torque

f. Equations

g. Matrix form: Where are the knowns?
Where are the unknowns?
h. Example solution for one position (Example 12-3)
   i. Masses
   ii. FBD's
   iii. Position vectors
   iv. Accelerations
   v. Forces (known)
   vi. Assemble matrix
   vii. Solve
   viii. Convert to polar
   ix. Coordinate systems
      Intra-link locations: ________________
      Placing values in matrix: _______________

11.5 Force Analysis of a Fourbar Slider-Crank Linkage
   a. Applicable to slider-cranks within internal combustion engines

   b. Equations for bars ___ and ___: same as before

   c. Equations for link ___.
      i. Angular and y accelerations are zero
      ii. Coulomb friction between links 3 and 4
      iii. F_p acts on link 4

   d. Eight remaining equations assembled into a matrix
C. Whence the information about displacement, velocity, and acceleration coming?
Lecture 5

V. Inverted Slider-Crank and Linkages with More Than Four Bars

A. Assignment
   See Lecture 4. Especially, the list-solve part.

B. Book
   11.6 Force Analysis of Inverted Slider-Crank
      a. Coriolis acceleration
         b. Free-body diagram (what's wrong with this picture?)
      c. Kinematic diagram
      
      d. How could one find the moment between links 3 and 4?
11.7 Force Analysis of Linkages with More Than Four Bars

a. Generalizations

b. Is there a limitation concerning the number of degrees of freedom for the mechanism?

c. What must be known about the mechanism?
VI. Shaking and Dynafour Program

A. Assignment: Problem 11-3, sentence 4, parts a and b (without Dynafour). Which mechanism has larger shaking force? Why? Using dynafour, reposition the centers of mass until you have reduced the shaking force by 50%. State the new position of the mass centers. Print out the forces for the original and new design, as well as the positions of the balance masses.

B. Book

11.8 Shaking Forces and Shaking Torque

a. Shaking force
   i. Definition: _______ of all forces acting on the ___________ ____________.
   ii. Equation

   iii. Example: Slider-crank, p. 469, Fig. 12-4 b.

   iv. Can you give an example of shaking force?

b. Shaking torque
   i. Definition: __________ torque felt by the ground plane.
   ii. Equation

   iii. Example, p. 469. How well does this fit our definition?

   iv. Engine tends to "shake" in rotation

c. Methods of limiting shaking
   i. Balancing (mass distribution)
   ii. Flywheels
   iii. Vibration-isolating engine mounts

11.9 Program Dynafour
a. Menu system and capabilities

b. Demonstration
VII. Energy Methods and Flywheels
   A. Assignment: Problem 11-4a.

   B. Book
      11.10 Linkage Force Analysis by Energy Methods
         a. Can solve for individual forces without Newtonian matrix approach
         b. Fourbar Linkage
            i. Diagram
   
         ii. Virtual work equation
   
         iii. Vector form
   
         iv. Expand to create scalar equation
   
         v. Additional information that must be provided: ______________ (on p. 547).
            (Newtonian method required only ______________.)
   
         vi. Solve one equation for $T_{12}$
   
         vii. Only works for input forces or torque, not for internal forces

11.11 Flywheels
   a. Reason for flywheels
      i. Torque fluctuations at the crank occur due to inertia distribution of mechanism
         (diagram: Figure 11-18, p. 548)
ii. The mechanism stores and delivers kinetic energy each cycle

iii. Maximum torque can be much larger than the __________ torque

iv. For most motors, an increased torque results in a ______________
    ________________, but we want speed to be constant

v. Speed and torque throughout a cycle can be smoothed out by
    ______________________________________________________________________
    ______________________________________________________________________
    ______________________________________________________________________

vi. Example: old JD tractor
   a) Slow ________
   b) _______ cylinders
   c) Large, visible flywheel keeps mechanism going, even through the
      ____________ portion of the cycle

b. Energy equations for a flywheel system.
   i. Assume a ______________ torque, $T_{avg}$ for now
   ii. FB Diagram and equation of equilibrium

iii. Look at speed as a function of _____________

iv. Resulting nifty integral

v. Interpretation of integral: the minimum and maximum speeds of the crank
   throughout the cycle are related to the area under the ______________
   between the orientations of maximum and minimum ______________.

vi. The energy the flywheel must store between its high-energy point and low-
    energy point is given by the difference in squared
    ____________________.
c. Example 11-5
   i. Torque profile: Figure 11-11
      a) Great variation in torque. Some torque is even negative, indicating
         ______________________
      b) How do we figure out how much energy is needed to be stored?
         ______________________ to find area under T-theta curve: units of ?
   ii. Why is minimum omega at B?
   iii. Why is omega largest at C?
   iv. Summation of areas under torque curve
VIII. Flywheels; Class Design Project

A. Assignment: Specify a flywheel $I$ for the mechanism in Problem P11-3a so that the coefficient of fluctuation $k$ is 0.05. Use your four-bar solver to produce the torque profile and confirm the effectiveness of your design, but compute the flywheel size by hand and show your work. Assume input speed to be the average. Show your work and record the computer program output to justify your answer.

B. Book

11.11 Sizing the flywheel (continued)

a. From last time: energy equation for flywheel

b. One can integrate area on the torque diagram for a constant-speed mechanism to arrive at the energy which must be stored by the flywheel (Example 11-5).

c. Once the energy is known,

\[ E = \]

d. Coefficient of fluctuation is defined as the ratio of \[ \text{__________ fluctuation to average ________} \]

\[ k = \]

\[ \text{omega}_{\text{ave}} = \]

e. Energy in terms of design parameters

\[ I_s = \]

f. $I_s$ is the moment of inertia of the \[ \text{__________ _______} \]. The flywheel would be just part of this.

g. $I_s$ small: \[ \text{__________ fluctuation, __________ k} \]
h. $I$, large: ________________ fluctuation, ____________ k

i. Benefit of flywheel, Figure 11-12. Much smoother ____________ output. (However, in the calculation of this curve, constant speed was still used as an assumption!)

j. Procedure:

Assume:                      Analyze:

Put on flywheel:             Re-analyze torques

11.13 Practical Considerations

a. Typical design procedure

   i. First-pass geometric design

   ii-iii. Figure mass, I, CG's

   iv. Dynamic force analysis

   v. Redesign parts to achieve satisfactory geometry, kinematics, and dynamics

b. By now, you see that this design is an iterative process!

c. Repeated design iterations--number crunching--is best done in the computer: TK solver, etc. Try to minimize "human handling" of the numbers: avoid errors due to pushing buttons repeatedly.

d. Which leads us to a discussion of the Class design project:
C. Design Problem
IX. Static Balance, Quiz 2

A. Assignment
   1. Problem 12-1 a,b.
   2. Problem 12-2 (suggestion: use a circular shape with one edge at the pin)

B. 12.0 Introduction
   a. Balancing modifying the mass distribution of a linkage to eliminate shaking forces and torques
   b. Accepted design practice is to balance all parts
   c. Static vs. dynamic balancing
   d. Balancing often must be done after manufacture

12.1 Static Balance
   a. Definition: Static balance is achieved when the sum of all the shaking forces (including D'Alembert) in the plane is equal to zero.
   b. Also known as single-plane balance
      i. Part must be relatively short in the axial direction
      ii. Examples: propellor, bike tire
   c. Vee-shaped link rotating at $\omega$
   i. Dynamic model: replace legs with concentrated masses at the center of mass of each leg
      a) Critical thought question: can you do this?
b) Implications on centripetal accelerations of masses

Since forces are a linear function of R, all mass may be placed at the "average" location.

c) Implications on angular acceleration of the link

d) The model is valid for balancing, but not for the calculation of __________________ ________________!

ii. Equation

a) Critical thought question: how does this differ from simply placing the center of mass at the center of rotation of the link?

b) Is there only one position at which the mass may be placed?

iii. Conversion to polar coordinates
a) Why leave in the negative signs?

b) The atan2 function: quadrant-specific. If

\[ \tan(\theta) = \frac{y}{x}, \text{ then} \]

\[ \text{atan2}(y, x) = \theta \quad \text{or sometimes} \]

\[ \text{atan2}(x, y) = \theta \]

\[ R_b = \]

\[ \mathbf{R}_b \mathbf{m}_b = \]

d. Example (if time allows)

C. Quiz 2 (30 min)
Lecture 10

X. Dynamic Balancing and Balancing of Fourbar Mechanisms

A. Assignment

1. Problem 12-5 a,b.

2. Extra credit problem. Starting from equations (12.8a), develop a method for perfectly balancing a slider-crank mechanism by specifying the mass center of the crank and of the connecting rod. Clue: for a slider-crank, \( l_4 = b_4 = \text{INF} \), and \( \phi_4 = 0 \).

B.

12.2 Dynamic Balance

a. Using static balance, you can balance in a plane, but what if you have a long shaft with unbalanced masses in several planes?

b. Can we avoid balancing each unbalanced plane separately?

c. Process known as dynamic balancing (also known as ________ - __________ balancing)

d. Dynamic balance: all reaction forces and moments must be zero

(appplies to moments in any plan which includes the shaft)

e. Dynamic balancing required in rotating parts with relatively long axial dimensions

i.

ii.

iii.
f. Balance by design first, then after manufacture, on a balancing machine

g. ______________ balance individual components before assembly, then dynamically balance the assembly (e.g. ___________ assembly)

h. Derivation

$$\sum F = 0$$

$$-m_1\overline{R}_1\omega^2 - m_2\overline{R}_2\omega^2 - m_3\overline{R}_3\omega^2 - m_4\overline{R}_4\omega^2 - m_5\overline{R}_5\omega^2 = 0$$

Two equations, four pairs of unknowns:

$$m_A \overline{R}_Ax + m_B \overline{R}_Bx =$$

$$m_A \overline{R}_Ay + m_B \overline{R}_By =$$

This gives force balance in the x,y directions; now we need moment balance.

$$\sum M_x = 0$$

$$\sum M_y = 0$$

Two more equations:
i. Note: you would use the equations from _________ first, since they allow design of \( m_b R_b \). Then you would use that value in the force equilibrium equations to find \( m_a R_a \).

j. Example: add masses in planes A and B to dynamically balance a long shaft with one unbalanced mass

12.3 Static Balancing the Fourbar Linkage

a. Static balancing means making the center of mass stationary

b. This is easy enough for a single link, but what about a fourbar linkage? The couple does not rotate about a fixed point.

c. However, there is a method to force the overall mass center of the linkage to be stationary

d. Berkof-Lowen method

e. Diagram of a fourbar
f. Mass center

g. Complex vector notation for convenience

h. Loop equation

i. Author gets to Equation (12.8b)

j. Position of CG3 ($\phi_3, b_3$) may be determined at will by the designer. Equations 12.8c and 12.8d specify the position-mass combination for the ________ (link 2) and the ______________ (link 4)

e.g.

k. Euler's identity

$$e^{i\phi} = \cos\phi + jsin\phi$$

l. Equations (12.8) could be rearranged so that the position of the CG of link 2 or 4 could be specified at will. Then the equations would specify the position-mass combinations of the other two. This must be done for the extra credit problem.

m. One link is free to be designed at will. Balancing can then be done using the other two links.
XI. Balancing with Dynafour; Correcting Imbalance; Engine Design

A. Assignment: Problem 12-6

B. 12.4 Effect of Balancing on Input Torque

a. Balancing the fourbar can ___________ the individual links and ___________ their individual moment of inertia

b. This can increase the magnitude of the fluctuations in the _____ _______ to a mechanism

c. It can also increase the __________ ____________

d. The next section gives and example of this

12.5 Balancing with Dynafour

a. You have already done one HW problem in which you used Dynafour for balancing a fourbar. Section 12.5 gives you another example to look at.

b. Overhead of animation of the example linkage (Figure 12-5)

c. Overhead of the force balance menu
   i. First: Balance, specify radii and masses on links 2 and 4
   ii. Then: 10 Point Forces, 8 Shaking Force on the Ground Plane directly after performing the balance (do not recalculate)
   iv. Explanation
      Unbalanced:
      Berkof:
      Net:

d. Unbalanced Forces (Figure 12-7)

e. Force on ground--balanced
   i. Magnitude change ____________________

f. Forces on ground plane points are __________ images (Figure 12-9, 12-10)
g. Maximum torque and torque variation (shaking torque) at the crank both
_________________ for this balanced linkage.

h. You can print out the balance locations-masses for complete balance
   i. Print, forces, shaking forces

13.6 Measuring and Correcting Imbalance
   a. Balanced design will not be built perfectly balanced due to
      ____________________________ ____________________.
   b. Example: tire will have imbalance due to imperfect shape
   c. Static balance of a tire
      i. Tire spun horizontally on cone
      ii. Weights placed on machine until tire spins level
      iii. Weights then attached to rim
      iv. Not best--tire has relatively long axial dimension
   d. Dynamic balance of tire
      i. Figure 12-12
         ii. Force transducer measures forces relative to the angle measured by the
             __________ __________ at a given speed
         iii. Computer determines necessary locations and masses
         iv. Planes and radius correspond to the inner and outer ______ ________.

13.0 Introduction to Engine Design
   a. Figure 13-1. Discuss the parts of an engine

13.1 Engine Design
   a. Multi-cylinder, vee engines very popular
   b. Especially v6, v8
   c. _______________ of the slider-crank mechanism greatly influence these
      configurations
d. First we will look at single-cylinder engines

e. Figure 13-3
   i. Reverse of pumps made out the same mechanism type
      ii. TDC= ____________________
      ii. BDC= ____________________

f. Four-stroke cycle (Figure 13-4)

g. Two-stroke engine (Figure 13-5) (Lawnboy)
   i. Not as efficient
      ii. Higher pollution (some unburnt fuel escapes)
      iii. Simpler

h. Program Engine has a built-in gas force curve

C. Don't forget: quiz next time, over Chapter 12
Lecture 12

XII. Slider-Crank Kinematics and Quiz 3

A. Assignment: Problem 13-1.

B. 13.2 Slider-Crank Kinematics

(Subtitled: Every mathematical technique you never thought you'd see again integrated into one problem)

(Sub-subtitled: Mental calisthenics to warm up for the quiz)

a. Overview
   
i. Get x, v, a of the slider (piston) as functions of time for a constant speed

ii. Diagram of slider-crank mechanism

iii. You can reason that the slider motion would be near sinusoidal, something like

\[ x = A + B \cos \omega t \]

iv. It turns out that its motion can be estimated even more closely by a Fourier series of the form

v. We will try to dust off as many rusty mathematical techniques as possible in order to derive this

b. Slider motion as a function of time
   
i. Law of cosines!

\[ l^2 = x^2 + r^2 - 2xr \cos \omega t \]

ii. Quadratic formula!!
\[ x^2 - x(2r \cos \omega) + (r^2 - 1^2) = 0 \]

\[
x = \\
= \\
= \\
=
\]

We could differentiate this with respect to time, but this doesn't give comprehensible interpretation.

iii. Binomial theorem!!!

Remember . . .

\[(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3\]

In general,

\[(a + b)^n = \]

If \(a = 1\) and \(n = 1/2\),

\[(1 + b)^{1/2} = \]

So

\[\sqrt{1 - \frac{r}{i} \sin^2 \omega} = \]

Then

\[x = r \cos \omega + \]
iv. Beloved trigonometric identities!!!!

\[
\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}
\]

v. This is our punchline, a truncated Fourier series!!!!!!

General form:

\[
y = \frac{a_e}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))
\]

can represent any periodic wave form exactly.

vi. Now that we have the displacement in terms of \( \omega t \), we can differentiate quite easily!

\[
x =
\]

\[
\dot{x} =
\]

\[
\ddot{x} =
\]

Note that the \( 2\omega \) terms are more significant in the velocity and acceleration equations.

vii. Interpretation: Figure 13-8

c. Preview of next time: Superposition

i. Constant \( \omega \) assumed

ii. Analyze forces based on gas forces alone

iii. Analyze forces based on inertia forces and torques
iv. Sum the effects of $i$, $ii$ and $iii$

C. Quiz (50 pt, about 20 minutes)
XIII. Gas Forces; Equivalent Masses

A. Assignment: Problems 13.3, 13.7

B. Overview of the next two sessions

1. Force and torque analysis by superposition on slider-crank
   a. Gas forces only
   b. Inertia forces only
   c. Summation

2. Gas force and torque--13.3--today

3. Equivalent lumped-mass models which greatly simplify computation of inertia forces--13.4 today

4. Inertia torques, total torques (13.5, 13.6)--next time

C. 13.3 Gas Force and Gas Torque

   a. First part of super position--consider as if the linkage is sitting still
   b. Goal: get the driving torque to the crank as a function of the gas force on the piston
   c. First: some definitions

      i. ______ _________: Force on the piston due to the exploding fuel-air mixture

      ii. Gas torque (driving torque $T_{g21}$): Torque of the crank on the ground due only to the gas pressure

      iii. Gas reaction torque $T_{g12}$ (in this case): the reaction torque felt by the crank due to torque exerted on ground. Causes the engine to "rock" when revved.

   d. Free-body diagrams of links
e. Gas torque in terms of $\phi$, $\omega t$

f. Simplify to terms of $\omega t$ only

g. Approximation
13.4 Equivalent Masses

a. How this fits into big picture
   i. Superposition requires gas and inertia forces to be calculated separately
   ii. We want to calculate inertia forces
   iii. We want to do this using an extremely simple lumped-mass model for the initial design stages

b. Our goal in this section
   i. Nearly-dynamically equivalent model
   ii. Lumped (point) masses
   iii. Masses concentrated at the pin joints
   iv. We will talk about the computation of the inertia forces next time

c. Model of the conrod (connecting rod)
   i. Complex motion--must be dynamically equivalent
      *equal _____________________________
      *equal _____________________________
      *equal _____________________________
   ii. Diagram of Conrod (13-10a)
iii. Diagram of the two-mass model

iv. Three equations for equivalence

v. For simplicity of analysis, we specify that one of the masses is at the _________ pin

vi. Because of this, the masses must be:

vii. Location of second mass at ________________ of ________________!

viii. Typical conrod
* CG toward crank
* Center of percussion very close to the crank pin
* Approximate model--just place 2nd mass at crank pin
* Design stage--CG not fully known anyway

d. Model of Crank
   i. Simple rotary motion--statically equivalent model
   ii. Only mass and center of gravity must be equal. We are not worried about angular acceleration.
   iii. This is called statically equivalent
   iv. Equations

v. One mass at stationary center (inconsequential), and one at the crank pin

e. Resulting model
   i. Diagram
   ii. Very simple: two lumped masses
   iii. Allows simple, approximate analysis for design calculations
   iv. Drawback: may have locked in an assumption concerning the shape and mass distribution of your conrod
XIV. Shaking; Total Torque; Flywheels

A. Assignment: Problems 13-9, 13-11

B. Overview

1. Use the simplified, lumped mass model from last time to figure the inertia forced and torques

2. Use ______________ to combine gas and inertia forces and torques to get their totals

3. Look at some results of total torque from program ENGINE

4. Tie the results into flywheel design

C. 13.5 Inertia and Shaking Forces

   a. Diagram of slider-crank

   b. Definition of total inertia force:

   c. Total inertia force

   d. Components of the total inertia force:

   e. Definition of shaking force:

   f. Components of the shaking force:
g. Program ENGINE computes shaking force (diagram)

13.6 Inertia and Shaking Forces

a. Definition of inertia torque

(I think that Norton has not followed his sign convention in this section, but I think that he gets the correct equation for shaking torque in the end)

b. Easiest way to get it (remember from Lecture 13?)

c. Expanded (Equation 13.15b)

d. Can you believe this trig identity?

e. Inertia torque as a function of frequency

Note that it has a third harmonic

f. Shaking torque

Compare this with Equation 12.15c, p. 474
He has corrected the problem with the sign convention by this

13.7 Total Engine Torque

a. Superposition

b. Program ENGINE computes this sum: example, Figure 13-15
i. Low vs. high speed, and increased effects of inertia torque

ii. Negative, as well as positive torques at high speed

iii. What would be the result of this?

* 

* 

13.8 Flywheels

a. Flywheel (or inertia on crankshaft) is mandatory on a _________________ engine

b. Procedure for designing a flywheel is the same as that of a fourbar linkage

* 

* 

* 

c. Average crank speed must be specified. One must consider the "worst" case, which occurs at _____ speed

d. Program engine will specify flywheel I for a user-supplied coefficient of _________________, k.
Lecture 15

XV. Pin Forces and Balancing

A. Assignments (last assignments before the Chapter 13 Quiz two class sessions from today)

1. Problem 13-19

2. Build your own one-cylinder, four-stroke engine using the ENGINE software.
   a. Use the English system of units
   b. Specify your choice of bore, stroke, crank-to-connecting rod ratio, and other parameters requested as you input information from the keyboard. Print out your engine information.
   c. Plot and print the shaking forces, torque, and crankpin forces at redline (5000 rpm).
   d. Perfectly balance the crank, and plot and print the shaking forces.
   e. Now overbalance the crank so that the maximum shaking force is as small as you can get it. Plot and print the shaking forces again.
   f. Use the software to help you size a flywheel such that the maximum torque of the motor is 1.5 times its average torque. Plot and print the torque curve. What moment of inertia, $I$, is required? What is the coefficient of fluctuation?

B. Overview

1. We have used ___________ to combine gas and inertia torques and forces.
2. Using ___________ models for the crank and the connecting rod

3. We now need to solve for the pin forces using superposition

4. We will still use the lumped-mass model, but will have to separate the mass which belongs to the __________ from that which belongs to the __________________________

5. We will look at the effect of each mass individually

6. Balancing the lumped-mass slider-crank model

C.

13.9 Pin Forces in the Single-Cylinder Engine

   a. Will look at inertia forces in pins due to each of our modeled lumped masses separately
b. Nomenclature conventions
   i. First subscript
      * g: due to gas forces

      * i: due to inertia forces

   ii. Second subscript
      * p: due to piston

      * w: due to conrod mass at wristpin

      * c: due to conrod mass at ___________

      * r: due to __________ mass at the crankpin

c. Goal: get pin forces $F_{41}$, $F_{34}$, $F_{32}$, and $F_{21}$

d. Gas forces were determined in Section 13.3

\[ F_{g14} = F_g \tan \phi_f \]

\[ F_{g32} = -F_{g34} = -F_g \hat{i} + F_g \tan \phi_f \hat{j} \]

e. Forces due to piston mass alone

   i. Free-body diagrams
ii. Forces are determined from right to left, since the crank speed is "set in stone"

iii. Summation of forces in the x direction on body 4

iv. Body 4, y direction

v. Force 3-4

vi. Link 3 is a two-force member

vii. Two forces on link 2

f. Forces due to conrod mass at the wrist pin

i. Free-body diagrams

ii. Similarly, the equations of equilibrium are found

where do each of the equations come from?

\[ F_{h_{441}} = -m_{3b}a_y \tan \phi \hat{j} \]

\[ F_{h_{441}} = F_{h_{441}} \]

\[ F_{h_{4432}} = -m_{3b}a_y \hat{j} \tan \phi + m_{3b}a_y \tan \phi \hat{j} \]

\[ F_{h_{4432}} = F_{h_{4432}} \]
g. Forces due to the conrod mass at the crank pin

i. Free-body diagram

ii. D'Alembert force

\[ F_{bc32} = -m_{3a}g_A \]

Above, body 3 is acting on body 2

h. Forces due to the crank mass at the crank pin

j. Summation (superposition) of forces

i. Force of piston on ground (what does each component come from?)

\[ F_{41} = F_{g41} + F_{p41} + F_{n41} \]

\[ = Eq.(14.20) \]

ii. Force on wristpin (what does each component come from?)

\[ F_{34} = F_{g34} + F_{p34} + F_{n34} \]

\[ = Eq. (14.21) \]

iii. Force on crankpin (what does each component come from?)

\[ F_{32} = \]

iv. Force on main pin (what does each component come from?)

\[ F_{21} = F_{32} + F_{n21} \]
k. Program ENGINE uses these equations to get the pin forces
   
i. Experiment around with it

   ii. Wristpin forces at medium speed can be smaller than at either low or high speeds

13.10 Balancing the Single-Cylinder Engine

a. You can perfectly balance the crank (for the lumped masses at the crank pin

   b. Equation

   c. Resulting shaking forces are all in the x direction (Figure 13-24)

   d. One can overbalance the crank to lessen the maximum shaking force.
      i. Some shaking force is now in the y direction

D. If time permits, go to lab and play with program ENGINE
XVI. Design Trade-offs; Discuss Design Project; Multicylinder Engines

A. Assignment: Continue to work on problems assigned last time, and get ready for the quiz next meeting

B.

13.11 Design Trade-offs

a. Conrod-crank ratio (l/r)
   i. Discuss
   ii. High: smooth-running (piston acceleration near \( \text{__________} \))
   iii. Low
      * More compact
      * Higher pin forces (cylinder walls, etc.)

b. Bore-stroke ratio (B/S)
   i. Definitions
      * Bore: \( \text{__________} \) \( \text{__________} \) of cylinder
      * Stroke: \( \text{__________} \) to \( \text{__________} \), or 2r
   ii. Trade-off
      \[ V = \frac{\pi B^2 S}{4} \]
   iii. Designs
      * Square: B/S = 1
      * Oversquare
         - B/S \( \text{____} \) 1
         - pancake
- High gas forces $\Rightarrow$ high pin forces

* Undersquare

- B/S ______ 1

- pencil

- larger stroke, inertia forces

iv. Production engines: $0.75 < \text{B/S} < 1.5$

c. Materials

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Why</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piston</td>
<td>Al alloy</td>
<td></td>
</tr>
<tr>
<td>Conrod</td>
<td>Cast or forged steel; Al in small engines</td>
<td></td>
</tr>
<tr>
<td>Wristpins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Engine blocks</td>
<td>Cast iron; Aluminum</td>
<td></td>
</tr>
</tbody>
</table>

c. Class Design Project

1. Go over revised problem statement

2. Report format

3. Help on equations for analyzing problem

4. Questions

D. 14.1 Multicylinder Engine Designs

a. Figure 14-1
b. Terms

i. Crankshaft

ii. Crank throw: each cylinder's ________ on the crank ________

iii. Phase angle on crank

iv. Banks

E. Review questions from class, if time permits
   1. Piston position as function of $\omega t$
   2. Gas force and torque
   3. Equivalent lumped masses
   4. Inertia and shaking forces due to individual masses
   5. Superposition of 2 and 4
   6. Design trade-offs

F. Go look at an actual engine, if time permits
XVII. Multicylinder Engines; Crank Phase; Quiz 4

A. Assignment: 14-1a, first sentence

B. 14.1 Multicylinder Engines
   a. Figure 14-2 Inline 4
   
   b. Figure 14-4 BMW V 12
      i. Note vee angle: ___
      ii. Vee angle in V-8: 90°
   
   c. Radial engines
      i. WWII airplanes--Wright-Patterson
   
   d. What is your idea? Why not _____________________?

14.2 Crank Phase Diagram
   a. \[ \text{Delta phase angle is} \]

   b. Rules
      i. Diagram of four-bars on common crank
ii.

iii.

iv.

v.

c. Crank phase diagram

i. Sine wave

ii. Square representation

XVIII. Forces, Torques, and Moments in In-line Engines

A. Assignment: Problem 14-1

B. Overview

1. We have computed shaking (inertia) forces and torques in single-cylinder engines
2. Used lumped-mass approach
3. Now we will sum inertia forces and torques for inline multicylinder engines, accounting for crank phase angles
4. Since the crankshaft has length, we also will look at shaking moments (in a plane which includes the shaft)
5. We will come up with rules for eliminating shaking at fundamental frequency or at harmonics
6. Not concerned about gas forces today, just inertia shaking effects

C. 14.3 Shaking Forces in Inline Engines

a. Equation for shaking force in one crank-slider, with the crank balanced for $m_A$

b. Cranks are oriented with phase angle $\phi_i$
   i. $\phi_i$ is a lag
   ii. For crank $i$, the force in the reference direction (i-hat) is determined by
   iii. Diagram

c. Shaking force in reference direction
i. Note all forces in the ____ direction—the crank has been balanced for $m_A$

ii. Trig identity

d. Collecting summations

i. Same trig identity can be used on the fourth and sixth harmonics, which were not written

f. This simple rule illustrates the value of using lumped masses and Fourier series to approximate slider-crank dynamics (Ch. 13)

i. But what have we lost by making this assumption?

14.4 Inertia torque in engines

a. Review: what is the definition of inertia torque?

b. Inertia torque was given in equation 13.15e

i. Which harmonic is predominant?

C. Including all cylinders
d. Similar trigonometric identity used

e. Similar rules for canceling inertia torque for harmonics

f. Note the presence of the third harmonic

14.5 Shaking Moments in Inline Engines

a. What is meant by shaking moments?

b. Single cylinder engines had no shaking moments--all masses were modeled in one plane

c. Like the long shaft which needed __________ balancing, we now must deal with shaking moments in a plane which includes the shaft

d. Diagram

e. Summation of moments in the \(xz\) plane

f. Modify the equation from shaking forces to get one for shaking moments

   i. How is this different from the equation for shaking force?

g. Conditions for zero shaking moment
h. Can be extended to 4th and 6th harmonics as well

D. Thought questions
   1. What will each of these types of shaking look like in an automobile engine?
   2. Which do you think is easiest to balance out? Hardest?

E. Example
   1. Figure 14.7
   2. Table 14-1 (also balances torques)
   3. Table 14-2
XIX. Even Firing

A. Assignment: Problem 14-2a, (there are six possible firing orders). If none of these gives even firing, also specify a 4-cylinder engine design which does.

B. Overview

1. Last time: how crank phase angles influence inertia shaking forces and torques

2. This time: how firing order and phase angles influence gas forces and torques

C. 14.6 Even Firing

a. Last time we talked about crank phase for minimal inertia effects

b. This time, we will use the presupposition that crank angles should be oriented such that firing is ______________ over a ______________

   i. Even firing

   ii. Two-stroke engine (360° cycle)

   iii. Four-stroke engine (720° cycle)

   iv. Example: 4-cylinder, 4-stroke engine

But

Δϕ required for inertia effects to cancel in a 4-stroke engine conflicts with the Δϕ required for even firing

We will analyze how to achieve even firing and cancel as many inertia effects as possible
c. Delta power stroke angle $\Delta \psi$
   
   i. The actual phase between firings
   
   ii. Ideal 2-stroke engine
   
   iii. Ideal 4-stroke engine

d. Example
   
   i. Four stroke $0^\circ, 90^\circ, 180^\circ, 270^\circ$
   
   ii. Diagram of the shaft

   iii. Crank phase diagram
iv. Possible crank power stroke angles

v. Note: a two-stroke engine can achieve even firing for this crank configuration

e. Second example

i. We would rather sacrifice some balance (inertia effects) to achieve even firing

ii. Crank phases: 0°, 180°, 360°, 540°, 720°

iii. $\Delta \phi = 180^\circ$, not the best to cancel inertia

iv. Diagram of shaft

iv. Crank phase diagram

vi. Power strokes for even firing

vii. Trade-off: Table 14-3

* Force:
* Moments:
f. Primary moments canceled by mirror symmetry

   i. Schematic of engine, Figure 14-16

   ii. Crank phase diagram, Figure 14-17

   iii. Force and moment constants, Table 14-4

   iv. Demo unit
XX. Vee and Opposed Engines

A. Determine a feasible crank phase diagram and firing orders for a V-6 engine. Will shaking forces for frequencies $\omega$ and $2\omega$ be cancelled?

B. Overview

1. What was the conflict in the design of the inline 4 cylinder engine?

2. Today, we will see how the vee engine can overcome this conflict!

C. 14.7 Vee Engine Configurations

a. Vee engine schematic, Figure 14-20

i. Vee angle $2\gamma$

ii. Unit vectors $l^\gamma$ and $r^\gamma$

iii. Shaking forces in these two directions

iv. Still using assumption of balanced crank

b. Shaking forces

i. Forces in the directions of the two banks, Equation 14.10d

$$\Theta$$ measured from the x-axis (halfway between banks)

ii. Second trigonometric identity leads to Equation 14.10h
iii. Criteria for zero shaking forces are the same as for inline engines

iv. Resolve shaking forces in the x,y directions (Equation 14.10)

c. Shaking moments

   i. Directions of the $m^\wedge$, $n^\wedge$ unit vectors

   ii. Equations 14.11 a,b

   iii. Similar criteria for zero shaking moments as in the inline engine

   iv. Resolve into x, y directions

d. Inertia torques

   i. Equation, Eq. 14.12a,b
ii. Sufficient conditions for zero inertia torque

iii. Implication: this can be achieved by other means

e. Gas torque
   i. Equation 14.13

f. Conrod length
   i. Two conrods per throw

   ii. Vee engine shorter than inline

   iii. Vibrational modes of the crankshaft--best if it is short and stiff, in order to drive these frequencies up

g. Example engine: V8
   i. Why not use 0, 180, 180, 0° for each bank?

   ii. Crank phase angle and power phase angles for V8

   iii. We can get both the balancing benefits of the 90° crank phases and even firing with a V8 engine

   iv. Crank phase angles of $\phi_i = 0, 90, 180, 270°$ do not give mirror symmetry
v. $\phi_i = \ldots$ does allow mirror symmetry, cancelling out inertia moments, except for the primary ($\omega$). We can cancel that another way.

vi. Crank phase diagram (Figure 14-23)

vii. Note the vee angle $2\gamma =$

In general, $2\gamma =$

where m is an integer, for even firing

viii. Note both ______ firing and cancelled ________ effects (desirable crankphase angles) are possible with this engine

ix. Firing usually alternates between left and right banks--why?

x. Force and moment balance state of example engine. Table 14-5.

14.8 Opposed Engine Configurations

a. What would be the ideal vee angle of a V4?

b. Possible crank phase diagram of a V4

c. Can you say, "v6"?
XXI. Balancing Multicylinder Engines; Program Engine

A. Assignment:

1. Make a chart comparing the advantages and disadvantages of the following engines: V8, V6, Inline 4, Inline 6

2. Work on the class design project

B. Overview

1. Cover various topics in balancing multicylinder engines

2. Go to lab and use program engine

C.

15.9 Balancing Multicylinder Engines

a. General Statements

i. Two-stroke engines with even firing are balanced for shaking forces except for frequencies nω

ii. Four-stroke with even firing balanced for shaking forces except for what frequencies?

iii. Same shaking moments are cancelled if ________ ________ is used

b. Characteristics of specific engines

i. Inline engine needs to be 6-cylinder in order to be balanced through the fourth harmonic

ii. Inline 6 (0, 240, 120, 120, 240, 0°) has zero shaking forces and moments through the 4th harmonic and through the 3rd harmonic for the inertia ________ (there are reasons why popular engines are popular)
iii. V12 is two inline 6's on a single crank. Has these same balance properties.

iv. V8 has balanced primary and secondary forces, torques, and moments, except for the primary ________ (Table 14-5).

v. V6 has unbalanced primary and secondary moments.

Vee angle should be \( m \times 120^\circ \) for even firing, where \( m \) is an integer.

Sometimes \( 60^\circ \) or \( 90^\circ \) to reduce width and to reuse ________ made for V8's

Runs rough unless connecting rod splayed by \( 30^\circ \) on crank pin.

c. Flywheels

i. Used to smooth out unbalanced inertia torque.

ii. Total torque curve (integral) must be used to design the flywheel.

d. Balance shafts

i. Unbalanced forces and moments can be balanced using pairs of ________-

__________ balance shafts (Figures 14-26,27).

ii. One pair must spin at each harmonic for each bank of cylinders--why?

iii. A pair of equivalent eccentricities on the two shafts results in an oscillating force in a single direction which is used to cancel shaking force.

iv. If the pair of equivalent eccentricities is offset, a moment may be created which cancels the engine's shaking moment.

v. At \( \omega \), one of the balance shafts may be the ________ itself.

vi. Special case:

V8 has only unbalanced moments in the first harmonic (\( \omega \)) (Table 14-5) (until the fourth harmonic).

This primary moment rotates at a constant amplitude.
Therefore only one balance shaft is needed, and the __________ may be used!

How could this be achieved using the smallest amount of mass?

16 Program Engine

a. We will go to the computer lab and work on the engine design exercise provided
Lecture 22

XXII. Program Engine Mini-lab and Quiz

A. Program engine minilab

1. See handout.

B. Quiz
XXIII. Lumped-Parameter Models

A. Overview

1. We wish to use lumped-parameter models to describe the dynamics of cam-follower systems

2. Figure 15-6

3. Reduce multi-degree-of-freedom model to an approximate single-degree-of-freedom model

4. We will use second-order linear differential equations for the most part. We will analyze the response of the sdof system next time

B. 15.1 Lumped-Parameter Dynamic Models

a. Simple lumped-parameter dynamic model of a follower (Figure 15-1)

b. Free-body diagram

c. Equation

d. Convert to terms of modeled parameters

e. Particular solution
f. Various types of damping
   
i. Coulomb
   
ii. Viscous (linear)
   
iii. Quadratic
   
iv. Linear approximation

15.2 Equivalent systems

   a. Talk about Figure 15.6

   i. Components are not rigid
   
   ii. A model may be built out of springs, masses, and dampers.
   
   iii. Levers
   
   iv. How do we combine these components?

   b. Types of variables
i. Through: force. What would be an electrical analogy?

ii. Across: voltage. What would be the mechanical analogy?

c. Types of connections
   i. Series: components have the same force (__________ variable)
   
   ii. Parallel: components have the same across variable (displacement, velocity)

d. Combining springs
   i. Series

   ii. Parallel

e. Combining dampers
   i. Series

   ii. Parallel

f. Combining masses
   i. If they have the same displacement, they are parallel

   ii. Summation

g. Levers
   i. Goal: eliminate lever to get an equivalent sdof model
ii. Keep spring--replace mass

iii. Keep mass--replace spring

iv. Dampers are similar to springs in this

h. Example 15-1
   i. Equivalent stiffnesses come from stiffness equations of components

   ii. Equivalent masses come from the masses of components. Sometimes the mass of a flexible member is shared between nodes

   iii. Masses are combined (to get the model to a sdof one). This is an approximation; why?

   iv. Springs, dampers, and finally the lever are reduced to the sdof equivalents

j. We will look at the dynamic response of this single-dof system next time. It will be the solution to a 2nd order ________________

k. Alternate way: we do not have to make the approximation of reducing the system to a single degree of freedom

   i. There is a dof for each mass node

   ii. Displacements $x_i$ of each node can be put into a vector

   iii. Matrix equation

   iv. Another form (state-space)
v. A system of several differential equations including the same number of variables intermingled

vi. Can solve with TK solver, matlab, simulink, etc.
XXIV. Dynamic Force Analysis of the Force-Closed Cam Follower

A. Assignment: work on class design project

B. Overview

1. We have learned how the lumped-parameter model of a cam-follower may be reduced to a single-dof spring-mass-damper system

2. Now we will analyze the response of that system to a forcing input

3. This leads to limitations as to how fast this system can run

C. 15.3 Dynamic Force Analysis of the Force-Closed Cam Follower

a. Free-body diagram

b. Equation of motion

c. Restated

d. Homogeneous solution ($F_c = 0$)
i. Overdamped case: \((\zeta > 1)\)

\[s_1, s_2 \text{ are real}\]

Superposed exponential decays

ii. Critically damped \((\zeta = 1)\)

\[s_1, s_2 \text{ are real and equal}\]

Response looks similar to overdamped case

iii. Underdamped case \((\zeta < 1)\)

Many physical systems, including most followers fit this description

Equation

e. Particular solution

i. Forcing function

ii. Assume the solution

iii. Amplitude and phase

iv. Plot of the particular response
f. Complete response = homogeneous + particular
XXV. Resonance; Force-Closed Cam Follower

A. Assignment: work on class design project

B. Overview: How does the forced response of the one-dof cam-follower model affect its design?

C. 15.4 Resonance

   a. Look at plot of forced response amplitude (Figure 15-6)

   b. Response amplitude is a function of frequency ratio and damping

   c. Resonance occurs at $\omega_f = \omega_n$

   d. Rule of thumb: Keep $\omega_f < \omega_n/10$ to avoid large response amplitudes. What would a large amplitude response look like?

   e. Second rule (non-sinusoidal motion)

      i. Sketch

      ii. Rise-time ratio

         Keep $\tau$ below 0.5

   f. Small $\tau$ is equivalent to large __________. This is good.
g. Interpretation

Rise time must be at least twice as large as the part of the free response cycle which represents one radian

h. Design ramification: design hardware so that $\omega_n$ is much higher than $\omega_f$

*Lighter
*Stiffer

j. Aside: some machines run above the resonance speed. They break through the resonance when the speed of the motor ramps up. Example?

15.5 Kinetostatic Force Analysis of the Force-Closed Cam Follower

a. Kinetostatics (inverse dynamics problem)

? $\Rightarrow$ ?

b. Inverse dynamical equation with a spring preload

c. $F_{pl}$ must be large enough to maintain contact with the cam at all times. Keep $F_c$ positive

d. $F_c$ goes negative--

i. Large impact forces

ii. Criterion for redline maximum RPM

e. Example 15-2

i. Cam displacement program discussed in Chapter 9

ii. Dynamic force $F_c$ for two preloads. Figure 15-10
iii. One results in follower jump, one does not

iv. Higher speed--higher preload needed

v. Smoothness of cam profiles will affect spring preload needed and the redline speed of the engine

vi. Fundamental law of cam design: ________ _______!
A. Assignment: Continue working of class design project

B. Overview

1. Last time we talked about a spring-preloaded (force-closed) cam-follower, and criteria for avoiding follower jump

2. This time we will talk about a follower design which eliminates the problem of follower jump: the form-closed cam-follower

3. Also calculating the camshaft torque from the cam force diagram!

C. 15.6 Kinetostatic Force Analysis of the Form-Closed Cam Follower

a. Diagram of the form-closed cam follower

   i. Grooved or track cam

   ii. Conjugate cams on a common shaft

b. Another name for the form-closed cam system (almost sure to be on the quiz): _____________________!

c. Equation for cam force

   i. What's missing?

   ii. Why is this significant?
d. Example with same cam profile as last time (Example 15-11)

   i. Eight segments
   
   ii. Four rises and falls
   
   iii. Dynamic force: Figure 15-12
   

   iv. How does the maximum dynamic force compare to that of the force-closed cam follower without follower jump?

   e. Advantages of the desmodromic cam-follower

      i. No resonance (no spring, except the compliance of the follower itself)
      
      ii. Can be operated at high speeds without fear of follower jump (used in race car engines)
      
      iii. Lower cam force and camshaft torque

   f. Disadvantages of the desmodromic cam-follower

      i. Must be clearance in cam track
      
      ii. Inner and outer surface must be precisely ground (not necessary in force-closed cam followers)
      
      iii. When cam force changes sign, follower roller goes to the other side of the groove. Its direction of rotation must reverse, and the impact on the opposite wall is called _____________ __________.

15.7 Camshaft Torque

   a. We have discussed camshaft torque as a function of angle
   
   b. We can use this to compute camshaft torque
   
   c. Power in = power out
   
   d. Comparison of camshaft torque for force-closed and desmodromic cam-followers (Figure 15-13)
e. Variations in torque cause small changes in $\omega$ which can be smoothed out with a flywheel on the camshaft (Program DYNACAM)
XVII. Polydyne Cams; Dynamic Force Measurement

A. What is left in the class

1. Class design project
2. Quiz next time
3. Study for final

B. Problem associated with valve trains

1. Follower _________
2. Why?
   a. Equation 15.6b
   b. Figure 15–6
3. Measurements of engine valves have shown that the motion of the valve may differ greatly from that of the cam profile
4. Is there a way one could account for the dynamic response when designing the cam profile?

C. 15.8 Polydyne cams

a. Definition
   i. Polynomial + dynamic
   ii. Polynomial motion profiles
   iii. The dynamics of the follower are considered in the design of the cam profile
b. Procedure

i. Start with a desired valve motion function

ii. Work backward through the differential equation for the response of the valve train

iii. Come up with the cam profile to make the desired valve motion happen

iv. Polynomial motion functions work well because they are easy to manipulate

c. Example from Section 15.7

i. Polynomial rise and fall had the lowest dynamic force

ii. Figure 15-14

d. A simpler solution to the valve train problem stated above

i. Overhead cam has a very stiff follower system which has a higher-frequency resonance

ii. Motion of the follower deviates less from that of the cam profile

iii. Examples, pp. 642-3. What type of follower?

BMW V-12 _____________

Opposed 6 _____________

e. Now let's look at the type of measurements which show the need for the polydyne or overhead cam

15.9 Dynamic Force Measurement

a. Frequency response function
b. Frequency response
   i. Figure 17-19, 1st ed. only
   ii. Resonances of the follower are apparent

c. Response curves as functions of time
   i. Figure 15-16
   ii. Higher-order follower dynamics show up in actual response curves

17.10 Practical Considerations  (good questions for a quiz; answer why for each one)

   a. Keep follower lift to a minimum
   b. Arrange follower spring to preload all pivots in a consistent direction
   c. Keep the duration of the rises and falls as long as possible
   d. Keep follower train stiffness low and mass high
   e. Keep lever ratios close to 1
   f. Keep camshaft itself stiff in torsion and bending
   g. Keep the cam pitch circle to a maximum
   h. Use low backlash gears in the camshaft drivetrain