Size of the Moon

Purpose:
To calculate the radius and mass of the moon and to understand the train of reasoning associated with our understanding of astronomical data.

Equipment:
Calculator
Ruler

In this lab we will determine the mass of the moon by two different methods. Each of these methods reflect the way data is collected concerning the cosmos.

Mass from the Moon’s Size
This first method uses parallax to determine the distance to the moon. Once the moon’s distance is known and its size, we will assume an average density for the moon.

1. Distance to the moon from parallax.
Take attachments number 1 – 3 to do this portion of the lab. Attachments one and two are views of the sky on April 16, 2002 at 8:35 AM GMT from two different spots on the earth. The one is where the moon is just setting and the other is where the moon is just rising. If you look at the two views, you will notice that the position of the moon is different with respect to the background stars. This difference in position is due to ‘parallax’.

From the diagram below you can see how parallax arises from the two views of the moon. When the moon is setting (1) the moon appears to be amongst one cluster of stars, while when it is rising (2) it appears to be amongst a different cluster of stars. Since we measure all star locations as angles, the parallax for the moon’s position will be just the angular distance between the two locations of the moon.

Once the parallax angle is known, a right triangle can be constructed with one leg of the triangle being the radius of the earth and the opposite angle, \( \theta \), being half of the parallax angle, \( \beta \).
Calculation:

a) Plot the moon’s location from star chart 1 on to star chart 2.

b) Measure the distance between Aldebaran and the central star of the Pleiades (Alcyone) on the star chart.
   \[ d_{aa} = \text{________________________ cm} \]

c) Measure the distance between the two positions of the moon on the star chart and record it below.
   \[ d_{\text{moon}} = \text{________________________ cm} \]

d) Using the third attachment, which is a star chart for the constellation Taurus, measure the distance between Aldebaran and Alcyone.
   \[ d_{sc} = \text{________________________ cm} \]

e) On this same star chart there are markings for declenation and right ascension, the coordinates for stars on the Celestial Sphere. Measure the distance between RA = 3h DEC 0° and RA = 3h DEC +10°. This measurement corresponds to 10° in the sky.
   \[ d_{10} = \text{________________________ cm} \]

f) Using these measurement determine the parallax angle for the moon. Notice this is just a process of applying conversion factors. We want to convert \( d_{\text{moon}} \) into an angle and we know that \( d_{aa} = d_{sc} \) and \( d_{10} = 10° \).
   \[ \beta = d_{\text{moon}} \left( \frac{d_{sc}}{d_{aa}} \right) \left( \frac{10°}{d_{10}} \right) = \text{______________°} \]

g) From parallax determine the angle opposite of the earth’s radius, \( \theta \).
   \[ \theta = \text{________________________°} \]

h) From trigonometry the ratio of opposite side to hypotenuse is the sine function. Using the diagram for the earth and moon the sine of \( \theta \) becomes the ratio of the earth’s radius, \( R_e \), to the earth-moon distance, \( R \).
   \[ \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{R_e}{R} \]
   The radius of the earth is \( R_e = 6.38 \times 10^6 \) m
   \[ R = \frac{R_e}{\sin \theta} = \text{________________________ m} \]
i) Determine the diameter of the moon by using measuring its size on the star chart.

\[ \text{\( w_{moon} = \)} \text{________________________}_\text{cm} \]

j) Convert this distance into an angle as we did before.

\[ \text{\( \alpha_{moon} = \)} \text{\( w_{moon} \left( \frac{\text{\( d_{rc} \)}}{\text{\( d_{aa} \)}} \right) \left( \frac{10^\circ}{10} \right) \)} \text{\( = \)} \text{____________________°} \]

k) The distance between the observer and the moon is \( r \), which is the adjacent side of the triangle. Again using trigonometry we have

\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{r}{R} \]

The observer distance is then

\[ r = R \cos \theta = \text{____________________m} \]

l) The diameter of the moon can now be determined by using the resolution formula from class. Remember, \( \alpha \) needs to be in terms of seconds for this formula to work.

\[ s = \frac{ra}{2.06 \times 10^5} = \text{____________________m} \]

m) Once you have the diameter of the moon the radius and volume can be calculated.

\[ r_{moon} = \frac{s}{2} = \text{____________________m} \]

\[ V = \frac{4}{3} \pi r_{moon}^3 = \text{____________________m}^3 \]

g) Assuming the average density of the moon is the same as the earth’s, calculate the mass of the moon. The average density of the earth is 5520 kg/m\(^3\).

\[ m = \rho V = \text{____________________kg} \]
Mass from a satellite’s orbit of the moon
On July 22, 1967 a satellite was placed in orbit around the moon. This satellite was named Explorer 35 and the attached data gives the position of Explorer 35 at 15 minute intervals. From this data plot the orbit of the satellite and determine the orbital period and distance of the semi-major axis.

\[ a = \ \text{___________________ Lunar Radii} \]

\[ P = \ \text{___________________ hours} \]

Convert the semi-major axis into meters and the period into seconds
(1 lunar radius = 1.738 \times 10^6 m) (1 hour = 3600 s)

\[ a = \ \text{___________________ Lunar Radii} \]

\[ P = \ \text{___________________ hours} \]

Combining Kepler’s Laws with Newton’s form of gravitational force we get

\[ P^2 = \frac{4\pi^2}{GM} a^3 \]

Solving for M gives (G = 6.67 \times 10^{-11} \text{ Nkg}^2/\text{m}^2)

\[ M = \frac{4\pi^2}{GP^2} a^3 = \ \text{___________________ kg} \]

Questions:
1. How do the two masses compare? Are they close?

2. Which measurement do you think is most accurate?

3. What are the assumptions that go into each of these calculations?