A new probe is to be sent to the moon via the Pegasus launch system. The Pegasus launch system allows a small rocket to be attached to the underside of a high altitude jet plane. Once the jet plane reaches maximum altitude, it fires the small rocket into space. We will assume that the probe has a mass of 125 kg, the jet can achieve speeds of 340 m/s and the maximum altitude the jet can reach is 18.0 km.

a) Just as the probe is released from the jet, what is its potential energy? 
(Assume U=0 at r = ∞)

The potential energy due to gravity is \( U = -\frac{GMm}{r} \) where \( M \) is the mass of the earth and \( r \) is the distance from the center of the earth to the probe.

\[
U = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{kg}^2 / \text{m}^2)(5.97 \times 10^{24} \text{ kg})(125 \text{ kg})}{(6.37 \times 10^6 \text{ m} + 1.8 \times 10^4 \text{ m})} = -7.79 \times 10^9 \text{ J}
\]

b) At this point, neglecting the rotation of the earth, what is the kinetic energy of the probe? 

Just after being dropped the probe is traveling at the jet's speed. The kinetic energy is then 

\[
K = \frac{1}{2}mv^2 = \frac{1}{2}(125 \text{ kg})(340 \text{ m/s})^2 = 7.23 \times 10^6 \text{ J}
\]

c) When the probe reaches the moon, what is its potential energy? 
(Ignore the fact that the moon has any gravitational effect.)

Using the formula from part a) and putting in the earth/moon distance gives 

\[
U = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{kg}^2 / \text{m}^2)(5.97 \times 10^{24} \text{ kg})(125 \text{ kg})}{(3.85 \times 10^6 \text{ m})} = -1.29 \times 10^8 \text{ J}
\]

d) When the probe reaches the moon, we want it to have a velocity of 900 m/s. How much work must the probe's rocket provide to do this?

The kinetic energy at the moon needs to be 

\[
K = \frac{1}{2}(125 \text{ kg})(900 \text{ m/s})^2 = 5.06 \times 10^7 \text{ J}
\]

Now using the energy-work theorem we can find the amount of non-conservative work done on the system. This non-conservative work comes from the rocket.

\[
W = \Delta K + \Delta U = K_2 - K_1 + U_2 - U_1
\]

\[
W = (5.06 \times 10^7 \text{ J}) - (7.23 \times 10^6 \text{ J}) + (-1.29 \times 10^8 \text{ J}) - (-7.79 \times 10^9 \text{ J})
\]

\[
W = 7.70 \times 10^9 \text{ J}
\]
e) Let's assume that all of the work done by the probe's rocket is done just after release from the high altitude jet. (Probe is at the initial potential energy level.) What is the velocity of the probe after the rocket has finished firing? Apply the work-energy theorem again where the potential does not change, then the velocity after the rocket burns all its fuel is
\[
W = K_f - K_i = \frac{1}{2}mv_f^2 - K_i
\]
\[
v_f = \sqrt{\frac{2(W + K_i)}{m}} = \sqrt{\frac{2(7.70 \times 10^9 J + 7.23 \times 10^6 J)}{125kg}} = 1.11 \times 10^4 m/s
\]

f) How long will it take the probe to reach the moon?  
(Hint: Kepler's 3rd Law) 
The ellipse formed by the probe's orbit has a semi-major axis which is half the sum of the earth-moon distance, the earth's radius, and the altitude of the probe when it is launched.
\[
a = \frac{1}{2}(3.85 \times 10^8 m + 6.37 \times 10^6 m + 1.8 \times 10^4 m) = 1.96 \times 10^8 m
\]
Using Kepler's 3\textsuperscript{rd} Law and calculating the time period we get
\[
T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2 (1.96 \times 10^8 m)^3}{(6.67 \times 10^{-11} N \cdot m^2 / kg^2)(5.97 \times 10^{24} kg)}} = 8.64 \times 10^5 s
\]
To get to the moon is half this time giving
\[
T = 4.32 \times 10^5 s = 120 hr = 5.0 days
\]

g) How does the rotation of the earth, the jet's flight direction and the gravitational effect of the moon affect the calculations of this problem? (Just explain, don't calculate.) 
If you fly the jet in the direction of the earth's rotation, you gain the velocity of the earth's rotation, thus giving a higher kinetic energy. If you consider the moon's gravitational attraction, you will gain kinetic some kinetic energy due to the loss of potential energy with respect to the moon as you approach it.