Chapter 38 Problem 53 [†]

Solution
How old is the archaeological find?

The half-life of carbon-14 is 5730 yr. Since the carbon-14 content is 34% of current day values, then

$$\frac{N}{N_0} = 0.34$$

Using the half-life radioactive decay equation

$$N = N_0 2^{-t/t_{1/2}}$$

Solve for t

$$\frac{N}{N_0} = 2^{-t/t_{1/2}}$$

Take the log base-2 of each side

$$\log_2\left(\frac{N}{N_0}\right) = -t/t_{1/2}$$

$$t = -t_{1/2}\log_2\left(\frac{N}{N_0}\right) = -(5730\ yr)\log_2(0.34)$$

Therefore,

$$t = -(5730\ yr)(-1.556) = 8916\ yr$$

Most of us do not have calculators that do log_2. Therefore, the more reliable way of solving this problem is to find the decay constant first.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730\ yr} = 1.210 \times 10^{-4}\ yr^{-1}$$

Now use the base-e model

$$N = N_0 e^{-\lambda t}$$

Divide by N_0 and take the natural log of both sides

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

Therefore,

$$t = \frac{-\ln\left(\frac{N}{N_0}\right)}{\lambda} = \frac{-\ln(0.34)}{1.210 \times 10^{-4}\ yr^{-1}} = 8916\ yr = 8900\ yr$$

[†]Problem from Essential University Physics, Wolfson