Chapter 32 Problem 41 †

Given

$I = 2.43 \times 10^{-45}\ kg \cdot m^2$
$f = 8.40 \times 10^{12}\ Hz$

Solution

Find the wavelength for the transition from $n = 0\ l = 3$ to $n = 1\ l = 2$ for KCl.

The rotational energy is given by the formula

$$E_{rot} = \frac{\hbar^2}{2I} l(l+1)$$

The change in rotational energy is then

$$\Delta E_{rot} = E_{l=2} - E_{l=3} = \frac{\hbar^2}{2I} 2(2+1) - \frac{\hbar^2}{2I} 3(3+1)$$

$$\Delta E_{rot} = \frac{\hbar^2}{2I}(6-12) = -\frac{6\hbar^2}{2I} = -\frac{3\hbar^2}{I}$$

Replacing $\hbar$ with $h/2\pi$ gives

$$\Delta E_{rot} = -\frac{3h^2}{4\pi^2 I}$$

The vibrational energy is given by the formula

$$E_{vib} = (n + \tfrac{1}{2})hf$$

The change in vibrational energy is

$$\Delta E_{vib} = E_{n=1} - E_{n=0} = (1 + \tfrac{1}{2})hf - (0 + \tfrac{1}{2})hf = hf$$

Adding both energies together gives

$$\Delta E = \Delta E_{vib} + \Delta E_{rot} = hf - \frac{3h^2}{4\pi^2 I}$$

Set this equal to the energy of a photon and solve for wavelength gives

$$E = \frac{hc}{\lambda}$$

$$hf - \frac{3h^2}{4\pi^2 I} = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{hf - \frac{3h^2}{4\pi^2 I}}$$

Simplify

$$\lambda = \frac{c}{f - \frac{3h}{4\pi^2 I}}$$

Substitute in the appropriate values

$$\lambda = \frac{3.00 \times 10^8\ m/s}{(8.40 \times 10^{12}\ Hz) - \frac{3(6.63\times 10^{-34}\ J\cdot s)}{4\pi^2(2.43\times 10^{-45}\ kg\cdot m^2)}}$$

$$\lambda = 3.58 \times 10^{-5}\ m = 35.8\ \mu m$$

If the rotation was not taken into account, the wavelength would have been 35.7 μm.

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† Problem from Essential University Physics, Wolfson