

Chapter 35 Problem 51 †

**Given**

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + U(x, y, z)\psi = E\psi$$

**Solution**

a) Show that  $\psi = A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L)$  is a solution to Schrodinger's equation for a 3-dimensional well.

Inside the 3-dimensional well the potential energy function is zero.

$$U(x, y, z) = 0$$

Take the second partial derivative of the function with respect to x, y and z.

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2}{\partial x^2} (A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L))$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-n_x^2 \pi^2}{L^2} (A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L)) = \frac{-n_x^2 \pi^2}{L^2} \psi$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2}{\partial y^2} (A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L))$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{-n_y^2 \pi^2}{L^2} (A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L)) = \frac{-n_y^2 \pi^2}{L^2} \psi$$

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2}{\partial z^2} (A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L))$$

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{-n_z^2 \pi^2}{L^2} (A \sin(n_x \pi x/L) \sin(n_y \pi y/L) \sin(n_z \pi z/L)) = \frac{-n_z^2 \pi^2}{L^2} \psi$$

Then the left side of Schrodinger's equation becomes

$$-\frac{\hbar^2}{2m} \left( \frac{-n_x^2 \pi^2}{L^2} \psi + \frac{-n_y^2 \pi^2}{L^2} \psi + \frac{-n_z^2 \pi^2}{L^2} \psi \right) = \frac{\hbar^2}{2m} \left( \frac{n_x^2 \pi^2}{L^2} + \frac{n_y^2 \pi^2}{L^2} + \frac{n_z^2 \pi^2}{L^2} \right) \psi$$

Comparing this to the right side of the equation, then Schrodinger's equation is satisfied as long as

$$E = \frac{\hbar^2}{2m} \left( \frac{n_x^2 \pi^2}{L^2} + \frac{n_y^2 \pi^2}{L^2} + \frac{n_z^2 \pi^2}{L^2} \right)$$

b) Verify that the energy relationship matches that of the textbook.

Remember

$$\hbar = \frac{h}{2\pi}$$

Then

$$E = \frac{(h/2\pi)^2}{2m} \left( \frac{n_x^2 \pi^2}{L^2} + \frac{n_y^2 \pi^2}{L^2} + \frac{n_z^2 \pi^2}{L^2} \right) = \frac{h^2}{8m\pi^2} \left( \frac{n_x^2 \pi^2}{L^2} + \frac{n_y^2 \pi^2}{L^2} + \frac{n_z^2 \pi^2}{L^2} \right)$$

$$E = \frac{h^2}{8m} \left( \frac{n_x^2}{L^2} + \frac{n_y^2}{L^2} + \frac{n_z^2}{L^2} \right) = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2)$$

---

†Problem from Essential University Physics, Wolfson