Chapter 35 Problem 42 †

Solution

Find the probability of finding a particle in the left-hand third of an infinite square well if it is in the ground state.

The normalized wave function for a particle in an infinite square well is

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

To find the probability integrate the square of the wave function over the interval of interest.

$$P(x) = \psi^2 dx$$

$$P(left\ third) = \int_0^{L/3} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)\right)^2 dx$$

$$P(left\ third) = \int_0^{L/3} \frac{2}{L} \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$P(left\ third) = \frac{2}{L}\int_0^{L/3} \frac{1}{2}\left(1 - \cos\left(\frac{2n\pi}{L}x\right)\right) dx$$

$$P(left\ third) = \frac{1}{L}\left(x - \frac{\sin\left(\frac{2n\pi}{L}x\right)}{2n\pi/L}\right|_0^{L/3}$$

$$P(left\ third) = \frac{1}{L}\left(\frac{L}{3} - \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{L}\frac{L}{3}\right) - 0 + \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{L}0\right)\right)$$

$$P(left\ third) = \frac{1}{L}\left(\frac{L}{3} - \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{3}\right) - 0 + 0\right)$$

$$P(left\ third) = \frac{1}{3} - \frac{1}{2n\pi}\sin\left(\frac{2n\pi}{3}\right)$$

Since we are interested in the ground state, $n = 1$. Then

$$P(left\ third) = \frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right)$$

$$P(left\ third) = \frac{1}{3} - \frac{1}{2\pi}\frac{\sqrt{3}}{2} = 0.196$$

There is a 19.6% chance of finding the particle in the left third of the box. This is true for the right third too. Therefore, the probability of finding it in the middle third is 60.8%.

† Problem from Essential University Physics, Wolfson