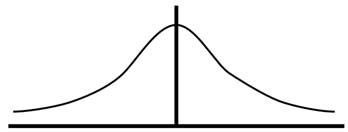
Chapter 35 Problem 13 †



Given
$\psi = Ae^{-x^2/a^2}$

Solution
a) Where is the particle most likely to be found?

Since the function is symmetric about zero, the particle is most likely to be found at zero. Mathematically the most likely place to be found corresponds to the position expectation value defined as

$$< x >= \int_{-\infty}^{+\infty} x\psi^2 dx$$

$$< x >= \int_{-\infty}^{+\infty} x(Ae^{-x^2/a^2})^2 dx = \int_{-\infty}^{+\infty} xA^2 e^{-2x^2/a^2} dx$$

$$< x >= A^2 \int_{-\infty}^{+\infty} xe^{-2x^2/a^2} dx$$

To solve, do a u-substitution of $u = -2x^2/a^2$. Then

$$\frac{du}{dx} = -4x/a^2$$

and

$$-a^2 du/4 = xdx$$

Now

$$< x >= A^2 \int_{x=-\infty}^{x=+\infty} e^u(-a^2/4)du$$

$$< x >= -A^2a^2/4(e^u|_{x=-\infty}^{x=+\infty}$$

$$< x >= -A^2a^2/4(e^{-2x^2/a^2}|_{-\infty}^{+\infty}$$

$$< x >= -A^2a^2/4(e^{-2(+\infty)^2/a^2} - e^{-2(-\infty)^2/a^2}) = 0$$

b) Find where the probability per length is half the maximum value.

The probability goes as the square of the probability function. The maximum value is at $x = 0$. So the half max value is where

$$\frac{1}{2} = \frac{\psi(x)^2}{\psi(0)^2}$$

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†Problem from Essential University Physics, Wolfson

$$\frac{1}{2} = \frac{A^2(e^{-x^2/a^2})^2}{A^2}$$

$$\frac{1}{2} = e^{-2x^2/a^2}$$

$$\ln\left(\frac{1}{2}\right) = -2x^2/a^2$$

$$-\ln 2 = -2x^2/a^2$$

Take the square root and solve for x.

$$\frac{a^2 \ln 2}{2} = x^2$$

$$x = \pm a\sqrt{\frac{\ln 2}{2}}$$