Chapter 33 Problem 73 †

Given

$m_e = 511\ keV$
$L' = 60\ cm$
$L = 57\ cm$

Solution

Find the velocity of the electrons and the accelerating potential difference to make the television work.

The length contraction from the perspective of the electron is

$$L = L'/\gamma = L'\sqrt{1 - v^2/c^2}$$

Solving for velocity gives

$$L^2/L'^2 = 1 - v^2/c^2$$
$$v^2/c^2 = 1 - L^2/L'^2$$
$$v = c\sqrt{1 - L^2/L'^2}$$

Substituting in the appropriate values gives

$$v = c\sqrt{1 - (57\ cm)^2/(60\ cm)^2} = 0.312c$$

The accelerating potential difference is really the voltage difference experienced by the electron as it accelerates. The potential difference must provide the change in kinetic energy.

$$\Delta K = -\Delta U = -q\Delta V$$

Since the electron is beginning at rest, then

$$\Delta V = -\frac{K}{q_e}$$

Now use the relativistic equation for kinetic energy

$$K = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right)mc^2$$

The rest mass energy of the electron is 511 keV. Instead of converting this to joules, we will keep these units. Substitute in the velocity of $0.328c$ and we get

$$K = \left(\frac{1}{\sqrt{1-(0.312c/c)^2}} - 1\right)(511\ keV) = (0.0525)(511\ keV) = 26.8\ keV$$

Then the accelerating potential must be

$$\Delta V = -\frac{26.8\ keV}{q_e}$$

Remember that an electron volt is the energy that is required to move the charge of the electron through one volt of potential difference. Dividing an electron volt by the charge of the electron will give the voltage. Therefore,

$$\Delta V = 26.8\ kV$$

You could do this problem by converting to joules and solving. You will get the same answer, but it will take a lot more work.

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† Problem from Essential University Physics, Wolfson