Chapter 33 Problem 46 †

Given
$x' = 25\ m$
$v_a = 0.65c$
$v_b = 0.50c$

Solution
a) Find the length of ship B in the earth's reference frame.

We will say the frame of reference, S', is moving with the space ship. The length contraction formula is then

$$x' = \gamma x$$

where x' is the length of the ship measured in the ship's reference frame and x is the ship's length in the earth's reference frame. Solving for x gives

$$x = \frac{x'}{\gamma} = \frac{x'}{\frac{1}{\sqrt{1-v^2/c^2}}} = \sqrt{1 - v^2/c^2}\,x'$$

Substituting in the provided values gives

$$x = \sqrt{1 - (0.50c)^2/c^2}(25\ m) = 21.7\ m$$

b) Find the length of ship B in ship A's reference frame.

First find the speed of ship B in A's reference frame. Let ship A's reference frame be S' and the earth's be S, then the velocity of ship B in earth's reference frame corresponds to u. Using

$$u' = \frac{u - v}{1 - uv/c^2}$$

where v is the speed of the earth's frame with respect to Ship A's frame and u' is the speed of ship B in A's reference frame. Recognize that A's velocity is in the positive direction and B's velocity is in the negative direction. Therefore, solving for u' gives

$$u' = \frac{(-0.5c) - (0.65c)}{1 - (-0.5c)(0.65c)/c^2} = -0.868c$$

The negative sign indicates that ship B is heading towards ship A at this speed. Now apply the length contraction formula given in part a.

$$x' = \sqrt{1 - (0.868c)^2/c^2}(25\ m) = 12.4\ m$$

Notice that I adapted the formula given above to the reference frame of ship A. We could go through a development of formulas going from S'' to S', but we would get the same result. Just remember that if two objects are moving relative to each other, the length will be shorter than if they were stationary with respect to each other.

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†Problem from Essential University Physics, Wolfson