Chapter 33 Problem 33 †

Given

$$t_0 = 20 \ yr$$

$$v = 0.95c$$

$$x' = 30 \; ly$$

Solution

Find the ages when the twins are united.

With respect to the earth's frame of reference the time of the trip will be the distance divided by the velocity.

$$t = \frac{d}{v} = \frac{30 \ ly}{0.95c} = 31.6 \ yr$$

Remember that a light year is the speed of light times one year. Therefore, a light year divided by the speed of light is one year. For the round trip the time is 63.2 years making twin A 83.2 yr old. For twin B the time will be shorter as determined by the time contraction formula

$$t' = \gamma \cdot t$$

Solving for the time in the moving space ship we have

$$t = \frac{t'}{\gamma} = \frac{t'}{\frac{1}{\sqrt{1 - v^2/c^2}}} = \sqrt{1 - v^2/c^2}t'$$

Using $t' = 63.2 \ yr$ which is the time for the whole trip twin B experiences

$$t = \sqrt{1 - (0.95c)^2/c^2}(63.2 \ yr) = 19.7 \ yr$$

Adding twenty years makes twin B $39.7 \ yr$.

[†]Problem from Essential University Physics, Wolfson