Chapter 33 Problem 15 [†]

Given
$x' = 50\ ly$
$v = 0.75c$

Solution
Find the distance between the two stars for those in the spaceship.

The Lorentz transform relating the observed displacements in the two coordinate frames is

$$x' = \gamma(x - vt)$$

Since we are considering the distance just as the trip begins we will assume that $t = 0\ s$. This now gives

$$x' = \gamma \cdot x$$

The primed variables are those in the coordinate frame at rest with the measured value while the non-primed variables are those in the coordinate frame of the moving observer. Therefore, x' is the measured distance between the stars in the star's rest frame and we want to find the distance, x, in the moving frame of the spaceship. Solving for x gives

$$x = \frac{x'}{\gamma} = \frac{x'}{\frac{1}{\sqrt{1-v^2/c^2}}} = \sqrt{1 - v^2/c^2}x'$$

Substituting in the provided values gives

$$x = \sqrt{1 - (0.75c)^2/c^2}(50\ ly) = 33.1\ ly$$

The distance to the spaceship is considerably shorter than the distance measured at rest with respect to the stars.

[†]Problem from Essential University Physics, Wolfson