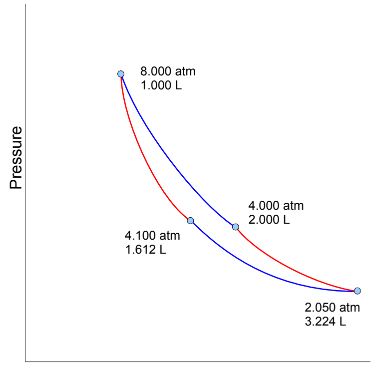
Chapter 19 Problem 42 †



Given

Figure 19-23

$P_1 = 8.000\ atm = 8.104 \times 10^5\ Pa$

$V_1 = 1.000\ L = 1.000 \times 10^{-3}\ m^3$

$P_2 = 4.000\ atm = 4.052 \times 10^5\ Pa$

$V_2 = 2.000\ L = 2.000 \times 10^{-3}\ m^3$

$P_3 = 2.050\ atm = 2.077 \times 10^5\ Pa$

$V_3 = 3.224\ L = 3.224 \times 10^{-3}\ m^3$

$P_4 = 4.100\ atm = 4.153 \times 10^5\ Pa$

$V_4 = 1.612\ L = 1.612 \times 10^{-3}\ m^3$

$n = 0.20\ mol$

Solution

a) Find the heat absorbed.

During the adiabatic processes (in red) no heat is exchanged with the surroundings. During the isothermal expansion (process 1 to 2) heat is absorbed and this is equal to the work since the temperature stays the same The work done during the isothermal expansion is

$$W = -nRT \ln\left(\frac{V_f}{V_i}\right) \tag{1}$$

From the ideal gas law we know that

$$PV = nRT \tag{2}$$

Replacing nRT in equation 1 with equation 2 gives

$$W = -P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) \tag{3}$$

$$W = -(8.10 \times 10^5\ Pa)(1.0 \times 10^{-3}\ m^3) \ln\left(\frac{2.0 \times 10^{-3}\ m^3}{1.0 \times 10^{-3}\ m^3}\right)$$

$$W = -561.5\ J$$

From the 1st law of thermodynamics the heat flow in is

$$\Delta Q = \Delta U - W = 0\ J - (-561.5\ J) = 561.5\ J$$

†Problem from Essential University Physics, Wolfson

$$Q_H = 561.5\ J$$

b) Find the heat rejected.

During the isothermal compression (process 3 to 4) heat is rejected and is equal to the work done on the gas. Using equation 3 we have

$$W = -P_3V_3 \ln\left(\frac{V_4}{V_3}\right)$$

$$W = -(2.077 \times 10^5\ Pa)(3.224 \times 10^{-3}\ m^3) \ln\left(\frac{1.612 \times 10^{-3}\ m^3}{3.224 \times 10^{-3}\ m^3}\right)$$

$$W = 464.2\ J$$

From the 1st law of thermodynamics the heat flow in is

$$\Delta Q = \Delta U - W = 0\ J - 464.2\ J = -464.2\ J$$

$$Q_C = 464.2\ J$$

c) Find the work done.

The work done is the difference between the heat absorbed and the heat rejected. This gives

$$W = Q_H - Q_C = 561.5\ J - 464.2\ J = 97.3\ J$$

d) Find the efficiency of the engine.

Efficiency is given by

$$e = \frac{W}{Q_H} \times 100\% = \frac{97.3\ J}{561.5\ J} \times 100\%$$

$$e = 17.3\%$$

e) Find the minimum and maximum temperature.

From the ideal gas law the temperature at 1 is

$$T_1 = \frac{P_1V_1}{nR} = \frac{(8.104 \times 10^5\ Pa)(1.0 \times 10^{-3}\ m^3)}{(0.20\ mol)(8.31\ J/mol \cdot K)} = 487.6\ K$$

The temperature at 3 is

$$T_3 = \frac{P_3V_3}{nR} = \frac{(2.077 \times 10^5\ Pa)(3.224 \times 10^{-3}\ m^3)}{(0.20\ mol)(8.31\ J/mol \cdot K)} = 402.9\ K$$

The Carnot efficiency is then

$$e = \left(1 - \frac{T_C}{T_H}\right) \times 100\% = \left(1 - \frac{402.9\ K}{487.6\ K}\right) \times 100\% = 17.4\%$$