Chapter 19 Problem 29 †

Given

$P = -750\ MW$ (This is negative because it is work done by the system.)
$T_i = 15°C = 288\ K$
$\Delta T = 8.5°C$
$flowrate = 2.8 \times 10^4\ kg/s$

Solution

a) Find the rate of heat extraction from the fuel.

From the first law of thermodynamics

$$\Delta U = \Delta Q + W$$

where ΔQ is the heat extracted from the fuel and ΔU is the heat gained by the power plant. During one cycle of the heat engine ΔU is removed and goes into heating the water. When considering the rate at which this process proceeds, the 1st law of thermodynamics becomes

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} + \frac{\Delta W}{\Delta t}$$

Solving for the rate of heat flow gives

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} - \frac{\Delta W}{\Delta t} \tag{1}$$

When heating up water the relationship between heat and temperature is

$$\Delta U = mc_{water}\Delta T$$

When considering the rate of heating of the water we need to consider how much water we are heating per second.

$$\frac{\Delta U}{\Delta t} = \frac{\Delta m}{\Delta t}c_{water}\Delta T = (flow\ rate)\,c_{water}\Delta T \tag{2}$$

Substituting equation 2 into 1 and remembering that the rate of work done is power we have

$$\frac{\Delta Q}{\Delta t} = (flow\ rate)\,c_{water}\Delta T - P$$

$$\frac{\Delta Q}{\Delta t} = (2.8 \times 10^4\ kg/s)(4184\ J/kg \cdot K)(8.5°C) - (-7.5 \times 10^8\ W)$$

$$\frac{\Delta Q}{\Delta t} = 1.75 \times 10^9\ W = 1.75\ GW$$

b) Find the efficiency of the power plant.

Using the rate at which work is done the rate at which energy is extracted we get an efficiency of

$$e = \frac{W}{Q_H} \times 100\% = \frac{\Delta W/\Delta t}{\Delta Q/\Delta t} \times 100\% = \frac{750MW}{1750MW} \times 100\%$$

† Problem from Essential University Physics, Wolfson

$$e = 42.9\%$$

c) Find the highest temperature.

Assuming the efficiency matches that of a Carnot engine, the efficiency is

$$e = \left(1 - \frac{T_C}{T_H}\right) \times 100\%$$

Solving for the hot temperature gives

$$T_H = \frac{T_C}{1 - \frac{e}{100\%}} = \frac{288K}{1 - \frac{42.9\%}{100\%}} = 504K$$

$$T_H = 231°C$$