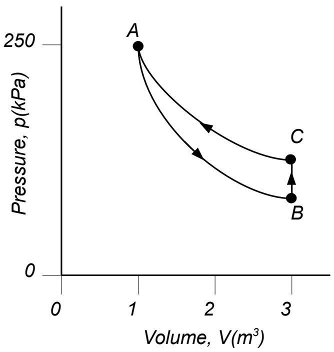
Chapter 18 Problem 51 †



Given
$\gamma = 1.67$
$P_A = 250\ kPa = 2.5 \times 10^5\ Pa$
$V_A = 1.00\ m^3$
$V_B = 3.00\ m^3$

Solution
a) Find the pressure at B.

The process from A to B is adiabatic. The relationship between pressure and volume is then

$$PV^\gamma = const.$$

The pressure and volume at A and B are then related by

$$P_A V_A^\gamma = P_B V_B^\gamma$$

Solving for the pressure at B gives

$$P_B = \frac{P_A V_A^\gamma}{V_B^\gamma} = P_A \left(\frac{V_A}{V_B}\right)^\gamma$$

$$P_B = (2.5 \times 10^5\ Pa)\left(\frac{1.0\ m^3}{3.0\ m^3}\right)^{1.67} = 3.99 \times 10^4\ Pa$$

b) Find the pressure at C.

The process from C to A is isothermal. There relationship between pressure and volume is then

$$PV = nRT = const.$$

The pressure and volume at C and A are then related by

$$P_C V_C = P_A V_A$$

Solving for the pressure at C gives

$$P_C = P_A\left(\frac{V_A}{V_C}\right) = (2.5 \times 10^5\ Pa)\left(\frac{1.0\ m^3}{3.0\ m^3}\right) = 8.33 \times 10^4\ Pa$$

†Problem from Essential University Physics, Wolfson

c) Find the net work done on the gas.

The work done during the adiabatic process is

$$W = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

$$W = -\frac{(2.5 \times 10^5\ Pa)(1.0\ m^3) - (3.99 \times 10^4\ Pa)(3.0\ m^3)}{1.67 - 1}$$

$$W = -1.94 \times 10^5\ J$$

The work done during the process B to C is $0\ J$ since the volume does not change.
The work done during the isothermal process is

$$W = -nRT \ln\left(\frac{V_f}{V_i}\right)$$

Since nRT is constant, we can replace it with $P_A V_A$.

$$W = -P_A V_A \ln\left(\frac{V_A}{V_C}\right) = (2.5 \times 10^5\ Pa)(1.0\ m^3) \ln\left(\frac{1.0\ m^3}{3.0\ m^3}\right)$$

$$W = 2.75 \times 10^5\ J$$

The total work is the sum of the work for all three processes.

$$W = -1.94 \times 10^5\ J + 0\ J + 2.75 \times 10^5\ J$$

$$W = 8.1 \times 10^4\ J$$