Chapter 38 Problem 23 [†]

Given
$t_{1/2} = 29\ years$

Solution
a) Find the time for 99% of the strontium-90 to decay.

The rate of radioactive decay is calculated using the equation

$$N = N_o e^{-\lambda t}$$

Taking this equation and solving for time gives

$$\frac{N}{N_o} = e^{-\lambda t}$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right)$$

The relationship between half-life and the decay constant is

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Solving for the decay constant gives

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

$$\lambda = \frac{\ln 2}{29\ yr} = 0.0239\ yr^{-1}$$

When 99% of the strontium-90 is decayed the ratio of present amount, N, to original amount, N_0, is 0.01. The time for 99% decay is then

$$t = \frac{-1}{0.0239\ yr^{-1}}\ln(0.01) = 193\ yr$$

b) Find the time for 99.9% of the strontium-90 to decay.

Using the same development as part a, use a ratio of $N/N_0 = 0.001$.

$$t = \frac{-1}{0.0239\ yr^{-1}}\ln(0.001) = 289\ yr$$

[†]Problem from Essential University Physics, Wolfson