Chapter 34 Problem 65 †

Given

$\Delta x = 20\ nm$
$m_e = 9.11 \times 10^{-31}\ kg$
$h = 6.63 \times 10^{-34}\ J \cdot s$

Solution

Find the minimum speed of an electron trapped in a quantum well of width 20 nm.

Begin with the Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

If you use the distinction that the direction of the momentum is unknown,then $\Delta p = p - (-p) = 2p$. See example 34.6 in the textbook. Applying this to Heisenberg's Uncertainty Principles gives

$$\Delta x 2p \geq \frac{h}{2\pi}$$

$$\Delta x p \geq \frac{h}{4\pi}$$

Often when Heisenberg's Uncertainty Principle is expressed, the distinction between left and right travelling particles is taken into account and the equation is

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

I will use the later form. Now momentum is mv. Therefore, the minimum velocity can not be known to be less than the uncertainty of the velocity. Therefore,

$$\Delta x m \Delta v = \frac{h}{4\pi}$$

$$\Delta v = \frac{h}{4\pi m \Delta x}$$

Substituting in the provided values gives

$$\Delta v = \frac{6.63 \times 10^{-34}\ J \cdot s}{4\pi(9.11 \times 10^{-31}\ kg)(20 \times 10^{-9}\ m)} = 2900\ m/s$$

†Problem from Essential University Physics, Wolfson