Chapter 19 Problem 63 †

Given

$C_P = a + bT + cT^2$
$a = 33.6 \ \frac{J}{mol \cdot K}$
$b = 2.93 \times 10^{-3} \ \frac{J}{mol \cdot K^2}$
$c = 2.13 \times 10^{-5} \ \frac{J}{mol \cdot K^3}$
$n = 2.0 \ mol$
$T_0 = 20\ °C = 293\ K$
$T_f = 200\ °C = 473\ K$

Solution

Find the entropy change as the gas heats isobarically.

Since the temperature changes as the gas is warming we must integrate to get the entropy change.

$$\Delta S = \int_1^2 \frac{dQ}{T} \tag{1}$$

The heat required to warm the gas is

$$\Delta Q = nC_P \Delta T$$

Therefore, for infinitesimal temperature changes

$$dQ = nC_P dT \tag{2}$$

Substituting 2 into 1 and integrating gives

$$\Delta S = \int_{T_0}^{T_f} \frac{nC_P dT}{T} = \int_{T_0}^{T_f} \frac{n(a + bT + cT^2)dT}{T}$$

$$\Delta S = \int_{T_0}^{T_f} n(a/T + b + cT)dT$$

$$\Delta S = n\left(a\ln(T) + bT + \tfrac{1}{2}cT^2\Big|_{T_0}^{T_f}\right.$$

$$\Delta S = n\left(a\ln(T_f/T_0) + b(T_f - T_0) + \tfrac{1}{2}c(T_f^2 - T_0^2)\right)$$

$$\Delta S = (2.0\ mol)((33.6\ \tfrac{J}{mol \cdot K})\ln(\tfrac{473\ K}{293\ K}) + (2.93 \times 10^{-3}\ \tfrac{J}{mol \cdot K^2})(473\ K - 293\ K)$$
$$+\tfrac{1}{2}(2.13 \times 10^{-5}\ \tfrac{J}{mol \cdot K^3})((473\ K)^2 - (293\ K)^2))$$

$$\Delta S = (2.0\ mol)(16.092\ \tfrac{J}{mol \cdot K} + 0.527 \tfrac{J}{mol \cdot K} + 1.468 \tfrac{J}{mol \cdot K})$$

$$\Delta S = (2.0\ mol)(18.09 \tfrac{J}{mol \cdot K}) = 36.2 \tfrac{J}{K}$$

†Problem from Essential University Physics, Wolfson