Chapter 35 Problem 53 †

Solution

Find the quantum state that corresponds to a 0.303 probability of finding the particle in the left-hand quarter of an infinite square well.

The normalized wave function for a particle in an infinite square well is

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

To find the probability integrate the square of the wave function over the interval of interest.

$$P(x) = \psi^2 dx$$

$$P(left\ quarter) = \int_0^{L/4} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)\right)^2 dx$$

$$P(left\ quarter) = \int_0^{L/4} \frac{2}{L} \sin^2\left(\frac{n\pi}{L}x\right) dx$$

$$P(left\ quarter) = \frac{2}{L} \int_0^{L/4} \frac{1}{2}\left(1 - \cos\left(\frac{2n\pi}{L}x\right)\right) dx$$

$$P(left\ quarter) = \frac{1}{L}\left(x - \frac{\sin\left(\frac{2n\pi}{L}x\right)}{2n\pi/L}\right|_0^{L/4}$$

$$P(left\ quarter) = \frac{1}{L}\left(\frac{L}{4} - \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{L}\frac{L}{4}\right) - 0 + \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{L}0\right)\right)$$

$$P(left\ quarter) = \frac{1}{L}\left(\frac{L}{4} - \frac{L}{2n\pi}\sin\left(\frac{2n\pi}{4}\right) - 0 + 0\right)$$

$$P(left\ quarter) = \frac{1}{4} - \frac{1}{2n\pi}\sin\left(\frac{n\pi}{2}\right)$$

If n is a multiple of 2, the sine function will give a value of zero and the probability will be 1/4 or 25% (as you would expect. If the value of n is odd, the following probabilities are calculated.

$$n = 1 \qquad P(left\ quarter) = \frac{1}{4} - \frac{1}{2(1)\pi}\sin\left(\frac{(1)\pi}{2}\right) = 0.0908 \qquad (9.08\%)$$

$$n = 3 \qquad P(left\ quarter) = \frac{1}{4} - \frac{1}{2(3)\pi}\sin\left(\frac{(3)\pi}{2}\right) = 0.303 \qquad (30.3\%)$$

$$n = 5 \qquad P(left\ quarter) = \frac{1}{4} - \frac{1}{2(5)\pi}\sin\left(\frac{(5)\pi}{2}\right) = 0.218 \qquad (21.8\%)$$

$$n = 7 \qquad P(left\ quarter) = \frac{1}{4} - \frac{1}{2(7)\pi}\sin\left(\frac{(7)\pi}{2}\right) = 0.273 \qquad (27.3\%)$$

The answer is $n = 3$. The additional calculations were included so you could see that as $n_{odd} \to \infty$, the probability $\to 0.25$.

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†Problem from Essential University Physics, Wolfson