Chapter 35 Problem 34 [†]

Given

$L = 25\ nm = 25 \times 10^{-9}\ m$

Solution

a) Find the wavelengths of the photons emitted for the transition $n = 2$ to $n = 1$.

For each part of this problem, it is necessary to calculate the energy of the transition followed by a calculation of wavelength. The energy of transition is

$$\Delta E = E_f - E_i = \frac{h^2 n_f^2}{8mL^2} - \frac{h^2 n_i^2}{8mL^2} = \frac{h^2}{8mL^2}\left(n_f^2 - n_i^2\right)$$

To eliminate repeated calculations, for this problem

$$\Delta E = \frac{(6.63 \times 10^{-34} J \cdot s)^2}{8(9.11 \times 10^{-31}\ kg)(25 \times 10^{-9}\ m)^2}\left(n_f^2 - n_i^2\right)$$

$$\Delta E = (9.65 \times 10^{-23}\ J)\left(n_f^2 - n_i^2\right) \tag{1}$$

Also the equation for calculating wavelength can be derived from the photon energy equation.

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} \tag{2}$$

Since energy lost by the electron is an energy gain by the photon, the answer using equation (1) will be negative, but we will substitute in a positive value for energy into equation (2).

Now let's solve part a) by substituting in the initial and final states into equation (1).

$$\Delta E = (9.65 \times 10^{-23}\ J)\left(1^2 - 2^2\right) = -2.895 \times 10^{-22}\ J$$

Substitute this energy into equation (2) to get the wavelength

$$\lambda = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^8\ m/s)}{2.895 \times 10^{-22}\ J} = 6.87 \times 10^{-4}\ m = 0.69mm$$

b) Find the wavelengths of the photons emitted for the transition $n = 20$ to $n = 19$.

Substitute the initial and final states into equation (1).

$$\Delta E = (9.65 \times 10^{-23}\ J)\left(19^2 - 20^2\right) = -3.76 \times 10^{-21}\ J$$

Substitute this energy into equation (2) to get the wavelength

$$\lambda = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^8\ m/s)}{3.76 \times 10^{-21}\ J} = 5.28 \times 10^{-5}\ m = 53\ \mu m$$

c) Find the wavelengths of the photons emitted for the transition $n = 100$ to $n = 1$.

Substitute the initial and final states into equation (1).

$$\Delta E = (9.65 \times 10^{-23}\ J)\left(1^2 - 100^2\right) = -9.65 \times 10^{-19}\ J$$

Substitute this energy into equation (2) to get the wavelength

$$\lambda = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^8\ m/s)}{9.65 \times 10^{-19}\ J} = 2.06 \times 10^{-7}\ m = 210\ nm$$

[†] Problem from Essential University Physics, Wolfson