

Chapter 35 Problem 30 †

Given

$$\lambda = 950 \text{ nm} = 950 \times 10^{-9} \text{ m}$$

Solution

What are the dimensions of a cubical box with an electron in it?

The energy of the photon emitted in the transition to the ground state is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^8 \text{ m/s})}{950 \times 10^{-9} \text{ m}} = 2.09 \times 10^{-19} \text{ J}$$

The energy states for a particle in a cubical box are

$$E_{n_x n_y n_z} = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2)$$

The ground state energy is

$$E_{111} = \frac{h^2}{8mL^2} (1^2 + 1^2 + 1^2) = \frac{3h^2}{8mL^2}$$

The next excited state has an energy of

$$E_{211} = \frac{h^2}{8mL^2} (2^2 + 1^2 + 1^2) = \frac{6h^2}{8mL^2}$$

Therefore the energy of transition is

$$E = E_f - E_i = E_{111} - E_{211} = \frac{3h^2}{8mL^2} - \frac{6h^2}{8mL^2} = -\frac{3h^2}{8mL^2}$$

This energy loss is an energy gain by the photon. Dropping the negative sign and solving for L gives

$$L^2 = \frac{3h^2}{8mE}$$

$$L = \sqrt{\frac{3h^2}{8mE}}$$

Substituting in the appropriate values gives

$$L = \sqrt{\frac{3(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(2.09 \times 10^{-19} \text{ J})}} = 9.3 \times 10^{-10} \text{ m}$$

†Problem from Essential University Physics, Wolfson