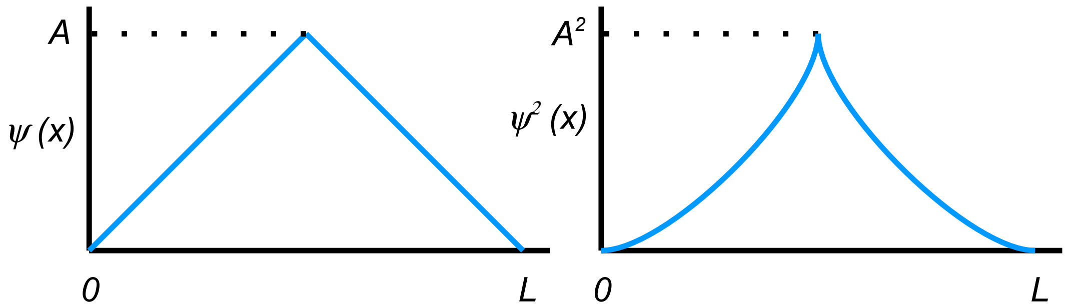
Chapter 35 Problem 12 †



Given

Solution

Find the value of A that normalizes the function.

If a wave function is normalized, the following should be true.

$$\int_{-\infty}^{\infty} \psi^2 dx = 1$$

Since the function for this curve is discontinuous, it must be defined for two intervals. The first interval goes from 0 to $L/2$. The curve has a slope of $A/(L/2)$ or $2A/L$. Since the y-intercept is zero, the equation for the first interval is

$$\psi_1(x) = \frac{2Ax}{L} \qquad 0 \le x < L/2$$

The second interval has a slope of $-2A/L$ and has a y-intercept of $2A$. Therefore, its equation is

$$\psi_2(x) = 2A - \frac{2Ax}{L} \qquad L/2 \le x \le L$$

Now square the wave function and integrate over its appropriate domain. Notice that the wave function has a value of zero outside of the given domain. Therefore,

$$\int_{-\infty}^{\infty} \psi^2 dx = \int_0^{L/2} \psi_1^2(x) dx + \int_{L/2}^{L} \psi_2^2(x) dx = 1$$

$$\int_0^{L/2} \left(\frac{2Ax}{L}\right)^2 dx + \int_{L/2}^{L} \left(2A - \frac{2Ax}{L}\right)^2 dx = 1$$

$$\int_0^{L/2} \left(\frac{4A^2x^2}{L^2}\right) dx + \int_{L/2}^{L} \left(4A^2 - \frac{8A^2x}{L} + \frac{4A^2x^2}{L^2}\right) dx = 1$$

$$\left(\frac{4A^2x^3}{3L^2}\bigg|_0^{L/2} + \left(4A^2x - \frac{8A^2x^2}{2L} + \frac{4A^2x^3}{3L^2}\bigg|_{L/2}^{L} = 1$$

$$\left(\frac{4A^2(L/2)^3}{3L^2} - 0\right) + \left(4A^2L - \frac{8A^2L^2}{2L} + \frac{4A^2L^3}{3L^2} - 4A^2(L/2) + \frac{8A^2(L/2)^2}{2L} - \frac{4A^2(L/2)^3}{3L^2}\right) = 1$$

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†Problem from Essential University Physics, Wolfson

$$\left(\frac{A^2L}{6} - 0\right) + \left(4A^2L - 4A^2L + \frac{4A^2L}{3} - 2A^2L + A^2L - \frac{A^2L}{6}\right) = 1$$

Notice several terms cancel each other and others easily combine without getting a common denominator.

$$\frac{4A^2L}{3} - A^2L = 1$$

Now get a common denominator and solve for A.

$$\frac{4A^2L}{3} - \frac{3A^2L}{3} = 1$$

$$\frac{4A^2L - 3A^2L}{3} = 1$$

$$\frac{A^2L}{3} = 1$$

$$A^2 = \frac{3}{L}$$

The value of A that normalizes the function is

$$A = \sqrt{\frac{3}{L}}$$