Chapter 34 Problem 41 †

Given
$\lambda_{max} = 660\ nm$

Solution
a) What is the temperature of the blackbody?

Using Wein's law

$$\lambda_{peak} T = 2.898 \times 10^{-3}\ m \cdot K$$

$$T = \frac{2.898 \times 10^{-3}\ m \cdot K}{\lambda_{peak}} = \frac{2.898 \times 10^{-3}\ m \cdot K}{660 \times 10^{-9}} = 4390\ K$$

b) What is the ratio of radiance at 400 nm compared to 700 nm?

The radiance formula for a blackbody radiator is

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

The radiance at 400 nm is

$$R(400\ nm, 4390\ K) = \frac{2\pi(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^8\ m/s)^2}{(400 \times 10^{-9}\ m)^5(e^{(6.63\times10^{-34}\ J\cdot s)(3.0\times10^8\ m/s)/((400\times10^{-9}\ m)(1.38\times10^{-23}\ J/K)(4390\ K))} - 1)}$$

$$R(400\ nm, 4390\ K) = 9.98 \times 10^{12}$$

The radiance at 700 nm is

$$R(700\ nm, 4390\ K) = \frac{2\pi(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^8\ m/s)^2}{(700 \times 10^{-9}\ m)^5(e^{(6.63\times10^{-34}\ J\cdot s)(3.0\times10^8\ m/s)/((700\times10^{-9}\ m)(1.38\times10^{-23}\ J/K)(4390\ K))} - 1)}$$

$$R(700\ nm, 4390\ K) = 2.05 \times 10^{13}$$

The ratio of the two radiance is then

$$\frac{R(400\ nm, 4390\ K)}{R(700\ nm, 4390\ K)} = \frac{9.98 \times 10^{12}}{2.05 \times 10^{13}} = 0.487$$

† Problem from Essential University Physics, Wolfson