Chapter 33 Problem 32 †

Given
$x' = 4\ ly$
$t' = 5\ yr$

Solution
a) According to the pilot of the spaceship, find the time it takes to get to the star.

The speed of the space ship can be determined through the kinematic formula

$$x' = v \cdot t'$$

Solving for v gives

$$v = \frac{x'}{t'} = \frac{4\ ly}{5\ yr} = 0.8c$$

Remember that a light year is the speed of light times one year. Therefore, a ly/yr is just the speed of light. To relate the time for the pilot to the time on the earth we must use the formula for time dilation

$$t' = \gamma \cdot t$$

Solving for the pilot's time, t, we have

$$t = \frac{t'}{\gamma} = \frac{t'}{\frac{1}{\sqrt{1-v^2/c^2}}} = \sqrt{1 - v^2/c^2}t'$$

$$t = \sqrt{1 - (0.8c)^2/c^2}(5\ yr) = 3\ yr$$

b) According to the pilot of the spaceship, find the distance to the star.

To relate the distance for the pilot to the distance for the earth we must use the formula for length contraction

$$x' = \gamma \cdot x$$

Solving for the pilot's time, t, we have

$$x = \frac{x'}{\gamma} = \frac{x'}{\frac{1}{\sqrt{1-v^2/c^2}}} = \sqrt{1 - v^2/c^2}x'$$

$$x = \sqrt{1 - (0.8c)^2/c^2}(4\ ly) = 2.4\ ly$$

†Problem from Essential University Physics, Wolfson