Chapter 33 Problem 16 †

Given
$v = 0.65c$
$x' = 5.8 \times 10^9\ km = 5.8 \times 10^{12}\ m$

Solution
a) Find the time to make the trip with respect to the earth's clock.

According to the earth's coordinate frame the velocity of the ship is

$$v' = 0.65c = 0.65(3.0 \times 10^8\ m/s) = 1.95 \times 10^8\ m/s$$

(Notice that v' is the velocity of the ship in the earth's frame. For part b we will use this velocity for v which is the velocity that Pluto is approaching the spaceship in the spaceship's reference frame.) The relationship between distance, velocity and time is

$$x' = v't$$

Solving for t' gives

$$t' = \frac{x'}{v} = \frac{5.8 \times 10^{12}\ m}{1.95 \times 10^8\ m/s} = 2.97 \times 10^4\ s$$

This time comes out to 8.25 hours.

b) Find the time to make the trip with respect to the spaceship's clock.

For the spaceship the distance to travel is calculated from the Lorentz transformation

$$x' = \gamma(x - vt)$$

where x' is the distance in the rest frame and x is the distance in the spaceship's moving frame. At the beginning of the trip, $t = 0$, the distance to travel is

$$x' = \gamma x$$

Solving for x gives

$$x = \frac{x'}{\gamma} = \frac{x'}{\frac{1}{\sqrt{1-v^2/c^2}}} = \sqrt{1 - v^2/c^2}x'$$

$$x = \sqrt{1 - (0.65c)^2/c^2}(5.8 \times 10^{12}\ m) = 4.41 \times 10^{12}\ m$$

Now that our variables are converted into the moving frame of the spaceship, the relationship between distance, velocity, and time is

$$x = v \cdot t$$

Solving for t gives

$$t = \frac{x}{v} = \frac{4.41 \times 10^{12}\ m}{1.95 \times 10^8\ m/s} = 2.26 \times 10^4\ s$$

This time comes out to 6.28 hours. Notice that you get the same value if you do the time dilation calculation directly from the answer in part (a) using the formula

$$t' = \gamma \cdot t$$

†Problem from Essential University Physics, Wolfson