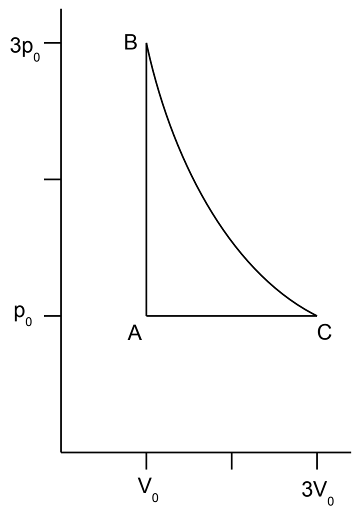
Chapter 19 Problem 56 †



Given
BC is isothermal

Solution
Calculate the entropy change for a full cycle and show that it equals zero for this reversible process.

5 moles is given for this problem, but as long as the number of moles stays constant, it doesn't matter how many moles you have. Therefore, I will use n to represent the number of moles. Break the cycle into 3 steps. A-B is isochoric. No work is done and heat flow is equal to the change in internal energy. Therefore, the heat flow is

$$\Delta Q = nC_V \Delta T$$

For infinitesimal temperature changes the heat flow is

$$dQ = nC_V dT$$

Assuming a monatomic gas,

$$dQ = \tfrac{3}{2} nRdT$$

The change in entropy is then

$$\Delta S_{AB} = \int_A^B \frac{dQ}{T} = \int_{T_A}^{T_B} \frac{\frac{3}{2}nRdT}{T} = \tfrac{3}{2} nR \ln\left(\frac{T_B}{T_A}\right)$$

$B - C$ is isothermal. Therefore heat flow equals the work done by the gas.

$$dQ = dW = PdV$$

Calculating the change in entropy we get

$$\Delta S_{BC} = \int_B^C \frac{dQ}{T} = \int_B^C \frac{PdV}{T}$$

†Problem from Essential University Physics, Wolfson

From the ideal gas law

$$\frac{P}{T} = \frac{nR}{V}$$

Therefore,

$$\Delta S_{BC} = \int_{V_B}^{V_C} \frac{nRdV}{V} = nR\ln\left(\frac{V_C}{V_B}\right) = nR\ln(3)$$

$C - A$ is isobaric. The heat flow in in this process is the change in internal energy plus the work done. When both of these are considered, the heat flow is

$$\Delta Q = nC_P\Delta T$$

For a monatomic gas,

$$dQ = \tfrac{5}{2}nRdT$$

The change in entropy is then

$$\Delta S_{CA} = \int_{C}^{A} \frac{dQ}{T} = \int_{T_C}^{T_A} \frac{\frac{5}{2}nRdT}{T} = \tfrac{5}{2}nR\ln\left(\frac{T_A}{T_C}\right)$$

Adding all the entropy changes together gives

$$\Delta S = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA}$$

$$\Delta S = \tfrac{3}{2}nR\ln\left(\frac{T_B}{T_A}\right) + nR\ln(3) + \tfrac{5}{2}nR\ln\left(\frac{T_A}{T_C}\right) \tag{1}$$

By the ideal gas law $T_A = \frac{P_AV_A}{nR}$,$T_B = \frac{P_BV_B}{nR}$, and $T_C = \frac{P_CV_C}{nR}$ Therefore

$$\frac{T_B}{T_A} = \frac{\frac{P_BV_B}{nR}}{\frac{P_AV_A}{nR}} = \frac{P_BV_B}{P_AV_A} = \frac{3P_0V_0}{P_0V_0} = 3$$

$$\frac{T_A}{T_C} = \frac{\frac{P_AV_A}{nR}}{\frac{P_CV_C}{nR}} = \frac{P_AV_A}{P_CV_C} = \frac{P_0V_0}{P_03V_0} = \frac{1}{3}$$

Now equation 1 becomes

$$\Delta S = \tfrac{3}{2}nR\ln(3) + nR\ln(3) + \tfrac{5}{2}nR\ln\left(\frac{1}{3}\right)$$

$$\Delta S = \tfrac{3}{2}nR\ln(3) + nR\ln(3) - \tfrac{5}{2}nR\ln(3) = 0$$