Chapter 19 Problem 43 †

Given
$m = 94,000\ kg$
$T_1 = 0°C = 273\ K$
$T_1 = 15°C = 288\ K$
$L_f = 334\ kJ/kg$

Solution
Find the entropy change when melting ice and then heating the water.

The heat flow into the ice during melting is

$$\Delta Q = mL_f = (94,000\ kg)(334\ kJ/kg)$$

$$\Delta Q = 3.14 \times 10^{10}\ J$$

The melting of the ice is at constant temperature. Therefore, the entropy change is

$$\Delta S = \frac{\Delta Q}{T} = \frac{3.14 \times 10^{10}\ J}{273\ K} = 1.15 \times 10^{8}\ J/K$$

Since the temperature changes as the water is warming we must integrate to get the entropy change.

$$\Delta S = \int_1^2 \frac{dQ}{T} \tag{1}$$

The heat required to warm water is

$$\Delta Q = mc\Delta T$$

Therefore, for infinitesimal temperature changes

$$dQ = mcdT \tag{2}$$

Substituting 2 into 1 and integrating gives

$$\Delta S = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S = (94,000\ kg)(4184\ J/kg \cdot K) \ln\left(\frac{288\ K}{273\ K}\right)$$

$$\Delta S = 2.10 \times 10^{7}\ J/K$$

$$\Delta S = \Delta S_{melt} + \Delta S_{warm}$$

$$\Delta S_{tot} = 1.15 \times 10^{8}\ J/K + 2.10 \times 10^{7}\ J/K = 1.36 \times 10^{8}\ J/K$$

$$\Delta S_{tot} = 136\ MJ/K$$

†Problem from Essential University Physics, Wolfson