Chapter 18 Problem 50 [†]

Given
$x_0 = 30\ cm = 0.30\ m$
$x_f = 17\ cm = 0.17\ m$
$T_0 = 20°C = 293\ K$
$\gamma = 1.4$

Solution
By how much does the temperature rise?

Assuming no heat is lost in the process (adiabatic process), all of the work done on the gas will go into internal energy as given by the 1st Law of Thermodynamics

$$\Delta U = Q - W = -W$$

As mentioned in the text, the relationship between volume and temperature for an adiabatic process is

$$TV^{\gamma-1} = constant$$

Therefore the initial and final values are related by the formula

$$T_0 V_0^{\gamma-1} = T_f V_f^{\gamma-1}$$

Solving for the final temperature gives

$$T_f = T_0 \left(\frac{V_0^{\gamma-1}}{V_f^{\gamma-1}} \right) = T_0 \left(\frac{V_0}{V_f} \right)^{\gamma-1}$$

Now volume equals displacement times cross-sectional area of the pump

$$V_0 = Ax_0 \qquad and \qquad V_f = Ax_f$$

Therefore, the final temperature is

$$T_f = T_0 \left(\frac{Ax_0}{Ax_f} \right)^{\gamma-1} = T_0 \left(\frac{x_0}{x_f} \right)^{\gamma-1}$$

Substituting in the appropriate values gives

$$T_f = (293\ K) \left(\frac{0.30\ m}{0.17\ m} \right)^{1.4-1} = (293\ K)\,(1.765)^{0.4} = 368\ K$$

The change in temperature is then

$$\Delta T = T_f - T_0 = 368\ K - 293\ K = 75\ K$$

[†]Problem from Essential University Physics, Wolfson