Chapter 17 Problem 68 [†]

Given
$\beta = a + bT + cT^2$
$a = -6.43 \times 10^{-5}\ {}^{\circ}C^{-1}$
$b = 1.70 \times 10^{-5}\ {}^{\circ}C^{-2}$
$c = -2.02 \times 10^{-7}\ {}^{\circ}C^{-3}$

Solution
Find the temperature at which water has the greatest density.

The temperature dependence of a volume for a substance can be expressed as a polynomial function derived from experimental data. This function taken to the third order would look like the following

$$V = V_0(1 + \alpha T + \lambda T^2 + \gamma T^3)$$

Therefore, the maximum of the volume (giving the minimum of density) could be determined by taking the derivative with respect to temperature and set it equal to zero. The derivative is

$$\frac{dV}{dT} = V_0(\alpha + 2\lambda T + 3\gamma T^2)$$

Now the coefficient of volume expansion is defined to be

$$\beta = \frac{\Delta V/V}{\Delta T}$$

Given the polynomial expression above we have

$$\Delta V = V - V_0 = V_0(1 + \alpha T + \lambda T^2 + \gamma T^3) - V_0 = V_0(\alpha T + \lambda T^2 + \gamma T^3)$$

Varying T gives

$$\beta = \frac{\Delta V/V}{dT} = V_0(\alpha + 2\lambda T + 3\gamma T^2)$$

Notice that the definition of β is really the slope of the polynomial function from the empirical data. Therefore, to get the maximum volume for the water, set β equal to zero and solve

$$\beta = a + bT + cT^2 = 0$$

Using the quadratic formula

$$T_{maxV} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2c} = \frac{-(1.70 \times 10^{-5}) \pm \sqrt{(1.70 \times 10^{-5})^2 - 4(-6.43 \times 10^{-5})(-2.02 \times 10^{-7})}}{2(-2.02 \times 10^{-7})}$$

$$T_{maxV} = 3.97,\ 80.2^{\circ}C$$

Since the second value is outside of the range under which the approximation is valid, the maximum volume must occur at $3.97\ {}^{\circ}C$.

[†]Problem from Essential University Physics, Wolfson