# General Physics III 

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## Coefficient of Linear Expansion

## Purpose:

The purpose of this experiment is to measure the coefficient of linear expansion for various metals.

## Equipment:

1. The 70 cm long expansion base with a built-in dial gauge and thermistor.
2. Three metal tubes: steel, copper $(99.5 \% \mathrm{Cu}, 0.5 \% \mathrm{Te})$, and aluminum ( $98.9 \% \mathrm{Al}, 0.7 \% \mathrm{Mg}, 0.4 \% \mathrm{Si})$.
3. A foam insulator to avoid heat loss at the thermistor connection point.
4. A source of steam or hot water plus tubing with $1 / 4$-inch inner diameter.
5. A digital ohmmeter plus leads with banana plug connectors.
6. A small block to raise the end of the expansion base approximately 2 -inches and a container to catch the water as it drains out of the tube.

## Theory:

Most substances expand when heated through a normal temperature range. The change in length $(\Delta \mathrm{L})$ when a solid is heated is proportional to the initial length $\left(\mathrm{L}_{\mathrm{o}}\right)$ and to the change in temperature $(\Delta \mathrm{T})$, or $\Delta \mathrm{L}=\alpha \mathrm{L}_{\mathrm{o}} \Delta \mathrm{T}$. The constant $\alpha$ is called the average coefficient of linear expansion, and is defined as the fractional change in length for each degree change in temperature. The value of the coefficient of linear expansion depends on the material of which the solid is made. To determine the value of $\alpha$ for different materials, which is the purpose of this investigation, the equation can be solved for the coefficient $\alpha$, $\alpha=\frac{\Delta L}{L_{0} \Delta T}$ where $\mathrm{L}_{\mathrm{o}}$ is the initial length, $\Delta \mathrm{L}$ is the change in length that results from an increase in temperature, and $\Delta \mathrm{T}$ is the change in temperature in degrees Celsius. Note that the average coefficient of linear expansion has units of $\left(\mathrm{C}^{\circ}\right)^{-1}$.

For materials that are not isotropic, such as an asymmetric crystal for example, $\alpha$ can have a different value depending on the axis along which the expansion is measured. $\alpha$ can also vary somewhat with temperature so that the degree of expansion depends not only on the magnitude of the temperature change, but on the absolute temperature as well.

In this experiment, you will measure $\alpha$ for copper, aluminum, and steel. These metals are isotropic so that $\alpha$ need only be measured along one dimension. Also, within the limits of this experiment, $\alpha$


Figure 1 Measuring Tube Length does not vary with temperature.

## Procedure:

1. Measure L, the length of the copper tube at room temperature. Measure from the inner edge of the stainless steel pin on one end, to the inner edge of the angle bracket at the other end (see Figure 1). Record your results in Table 1.
2. Mount the copper tube in the expansion base as shown in Figure 2. The stainless steel pin on the tube fits into the slot on the slotted mounting block and the tube presses against the spring arm of the dial gauge.


Figure 2 Equipment Setup (Top View)
3. Use one of the provided thumbscrews to attach the thermistor lug to the threaded hole in the middle of the copper tube. The lug should be aligned with the axis of the tube, as shown in Figure 2, so there is maximum contact between the lug and the tube.
4. Place the foam insulator over the thermistor lug as shown in Figure 2.
5. Plug the leads of your ohmmeter into the banana plug connectors labeled THERMISTOR in the center of the expansion base.
6. Measure and record $\mathrm{R}_{\mathrm{rm}}$, the resistance of the thermistor at room temperature. Record these values in the table.
7. Use tubing to attach your steam generator to the end of the copper tube. Attach it to the end farthest from the dial gauge.
8. Use a book or a block of wood to raise the end of the expansion base at which steam enters the tube (a few centimeters is sufficient). This will allow any water that condenses in the tube to drain out. Place a container under the other end of the tube to catch the draining water.
9. Turn the outer casing of the dial gauge to align the zero point on the scale with the indicator needle.
10. Turn on the steam generator. As steam begins to flow, watch the dial gauge and the ohmmeter. When the thermistor resistance stabilizes, record the resistance ( $\mathrm{R}_{\text {hot }}$ ) in Table 1. Also record the expansion of the tube length $(\Delta \mathrm{L})$ as indicated by the displacement of the indicator on the dial gauge. (Each increment on the dial gauge is equivalent to 0.01 mm of tube expansion.)

Caution: Be careful not to burn yourself with the condensed hot water in the hot tube. Be especially careful when you remove it.
11. Repeat the experiment for the steel and aluminum tubes.

## NOTES ON TEMPERATURE MEASUREMENT

A thermistor's resistance varies reliably with temperature. The resistance can be measured with an ohmmeter, and converted to a temperature measurement using the conversion table provided on the expansion base and also on the back page of this manual. Although the relationship between temperature and resistance is NOT linear, a linear approximation can be accurately used to interpolate between table data with an accuracy of approximately $+0.2^{\circ} \mathrm{C}$.
The thermistor used to measure the tube temperature is embedded in the thermistor lug. Once thermal equilibrium has been reached, the heat is highly uniform along the length of the tube. The foam insulator is used to inhibit heat loss through the thermistor lug so the lug temperature closely follows the tube temperature. The insulator does not have any appreciable effect on the local temperature of the tube itself.

## Linear Approximation (AKA Linear Interpolation)

When measuring the resistance of the thermistor, set the multimeter scale to the lowest range which includes the measured resistance. By doing this you will get the most number of significant digits. Since chances are your measurement will not match the table at the end of the handout, you must do linear interpolation. This is a skill which most of you have not learned due to the advent of the calculator. Let's say you measure a resistance of $24,900 \Omega$. Looking at the table you find the resistances that bound this number and their associated temperatures.

|  | Resistance | Temperature |  |
| ---: | :---: | :---: | :---: |
| $\mathrm{R}_{1}$ | $25,390 \Omega$ | $57^{\circ} \mathrm{C}$ | $\mathrm{T}_{1}$ |
| R | $24,900 \Omega$ |  | T |
| $\mathrm{R}_{2}$ | $24,415 \Omega$ | $58^{\circ} \mathrm{C}$ | $\mathrm{T}_{2}$ |

Graphically you could plot these numbers, draw a straight line connecting the end points and then read the value of T that corresponds to R. Numerically we notice that the slope of the line connecting the end points is

$$
m=\frac{\Delta T}{\Delta R}=\frac{T_{2}-T_{1}}{R_{2}-R_{1}}
$$

The value of R is then

$$
T=m\left(R-R_{1}\right)+T_{1}
$$

For this example

$$
\begin{aligned}
& m=\frac{\Delta T}{\Delta R}=\frac{58^{\circ} \mathrm{C}-57^{\circ} \mathrm{C}}{24,415 \Omega-25,390 \Omega}=-1.0256 \times 10^{-3}{ }^{\circ} \mathrm{C} / \Omega \\
& T=\left(-1.0256 \times 10^{-3}{ }^{\circ} \mathrm{C} / \Omega\right)(24,900 \Omega-25,390 \Omega)+57^{\circ} \mathrm{C} \\
& T=57.50^{\circ} \mathrm{C}
\end{aligned}
$$

From the description given at the top of the page, you uncertainty of you result if $0.2{ }^{\circ} \mathrm{C}$.

Data Table: Coefficient of Linear Expansion of Metals

|  | Copper | Steel | Aluminum |
| :---: | :---: | :---: | :---: |
| $\mathrm{R}_{\mathrm{rm}}(\Omega)$ |  |  |  |
| $\mathrm{T}_{\mathrm{rm}}\left({ }^{\circ} \mathrm{C}\right)$ |  |  |  |
| $\mathrm{L}_{0}(\mathrm{~m})$ |  |  |  |
| $\mathrm{R}_{\mathrm{hot}}(\Omega)$ |  |  |  |
| $\mathrm{T}_{\mathrm{hot}}\left({ }^{\circ} \mathrm{C}\right)$ |  |  |  |
| $\Delta \mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ |  |  |  |
| $\Delta \mathrm{L}(\mathrm{m})$ |  |  |  |
| $\alpha\left({ }^{\circ} \mathrm{C}^{-1}\right)$ |  |  |  |
| Accepted value of $\alpha$ |  |  |  |
| Percent Error |  |  |  |

1. Use the Table at the end of this lab, or the one on the top of the expansion base, to convert your thermistor resistance measurements, $\mathrm{R}_{\mathrm{rm}}$ and $\mathrm{R}_{\text {hot }}$, into temperature measurements, $\mathrm{T}_{\mathrm{rm}}$ and $\mathrm{T}_{\text {hot }}$. Record your results in the data table.
2. Calculate $\Delta \mathrm{T}=\mathrm{T}_{\text {hot }}-\mathrm{T}_{\mathrm{rm}}$. Record the result in the data table.
3. Using the equation $\Delta \mathrm{L}=\alpha \mathrm{L} \Delta \mathrm{T}$, calculate $\alpha$ for copper, steel, and aluminum. Enter the results in the data table.
4. Look up the accepted values for the linear expansion coefficient for copper, steel, and aluminum, and enter them in the data table.
5. Calculate the percent error in each case and enter them in the data table.
6. Using differential error analysis the relationship between uncertainty in the expansion coefficient and other variables is given below. Since we are already using $\Delta T$ and $\Delta L$, I have modified the notation where the uncertainties of your measurements are $d L, d(\Delta T)$, and $d(\Delta L)$. The uncertainty of the expansion coefficient is $d \alpha$.

$$
\frac{d \alpha}{\alpha}=\frac{d L}{L}+\frac{d(\Delta T)}{\Delta T}+\frac{d(\Delta L)}{\Delta L}
$$

## DISCUSSION QUESTIONS

1. Compare the accepted values for $\alpha$ with your experimental values. Are your experimental results within the range you would expect from the uncertainties of your measurements?
2. Which metal gave you the smallest percentage error and the largest percentage error? Why might the type of metal affect your results?
3. After comparing the individual terms in your error analysis, which measurement contributed the most error? How would you reduce this error if you were to run the lab again?
4. From your result, calculate the coefficients of volume expansion for copper, aluminum, and steel? (i.e., $\Delta \mathrm{V}=\alpha_{\mathrm{vol}} \mathrm{V} \Delta \mathrm{T}$ ) (Check the text for how to accomplish this if you are not sure.)
5. What are some practical areas where the effect studied in this experiment would be very important?
6. How could you use a demonstration based on this experiment to illustrate a biblical truth? Include a scripture passage along with a short explanation of how you would make the connection.

THERMISTOR CONVERSION TABLE:
Temperature versus Resistance

| Res. <br> $(\Omega)$ | Temp. <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Res. <br> $(\Omega)$ | Temp. <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Res. <br> $(\Omega)$ | Temp. <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Res. <br> $(\Omega)$ | Temp. <br> $\left({ }^{\circ} \mathrm{C}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 351,020 | 0 | 95,447 | 26 | 30,976 | 52 | 11,625 | 78 |
| 332,640 | 1 | 91,126 | 27 | 29,756 | 53 | 11,223 | 79 |
| 315,320 | 2 | 87,022 | 28 | 28,590 | 54 | 10,837 | 80 |
| 298,990 | 3 | 83,124 | 29 | 27,475 | 55 | 10,467 | 81 |
| 283,600 | 4 | 79,422 | 30 | 26,409 | 56 | 10,110 | 82 |
| 269,080 | 5 | 75,903 | 31 | 25,390 | 57 | $9,767.2$ | 83 |
| 255,380 | 6 | 72,560 | 32 | 24,415 | 58 | $9,437.7$ | 84 |
| 242,460 | 7 | 69,380 | 33 | 23,483 | 59 | $9,120.8$ | 85 |
| 230,260 | 8 | 66,356 | 34 | 22,590 | 60 | $8,816.0$ | 86 |
| 218,730 | 9 | 63,480 | 35 | 21,736 | 61 | $8,522.7$ | 87 |
| 207,850 | 10 | 60,743 | 36 | 20,919 | 62 | $8,240.6$ | 88 |
| 197,560 | 11 | 58,138 | 37 | 20,136 | 63 | $7,969.1$ | 89 |
| 187,840 | 12 | 55,658 | 38 | 19,386 | 64 | $7,707.7$ | 90 |
| 178,650 | 13 | 53,297 | 39 | 18,668 | 65 | $7,456.2$ | 91 |
| 169,950 | 14 | 51,048 | 40 | 17,980 | 66 | $7,214.0$ | 92 |
| 161,730 | 15 | 48,905 | 41 | 17,321 | 67 | $6,980.6$ | 93 |
| 153,950 | 16 | 46,863 | 42 | 16,689 | 68 | $6,755.9$ | 94 |
| 146,580 | 17 | 44,917 | 43 | 16,083 | 69 | $6,539.4$ | 95 |
| 139,610 | 18 | 43,062 | 44 | 15,502 | 70 | $6,330.8$ | 96 |
| 133,000 | 19 | 41,292 | 45 | 14,945 | 71 | $6,129.8$ | 97 |
| 126,740 | 20 | 39,605 | 46 | 14,410 | 72 | $5,936.1$ | 98 |
| 120,810 | 21 | 37,995 | 47 | 13,897 | 73 | $5,749.3$ | 99 |
| 115,190 | 22 | 36,458 | 48 | 13,405 | 74 | $5,569.3$ | 100 |
| 109,850 | 23 | 34,991 | 49 | 12,932 | 75 |  |  |
| 104,800 | 24 | 33,591 | 50 | 12,479 | 76 |  |  |
| 100,000 | 25 | 32,253 | 51 | 12,043 | 77 |  |  |

