Chapter 7 Problem 49 †

Given
$q = 5.00 \times 10^{-3}\ C$
$D = 2.0\ m$
$k = 8.99 \times 10^{9}\ Nm^2/C^2$

Solution
a) Find the potential just above the surface of the Van de Graaff.

For a spherically shaped object, the electric field is

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

The voltage difference between two points is

$$\Delta V = -\int_{r_0}^{r_f} \vec{E} \cdot d\vec{r}$$

$$\Delta V = -\int_{r_0}^{r_f} \left(\frac{kq}{r^2}\hat{r}\right) \cdot dr\hat{r} = -kq \int_{r_0}^{r_f} \frac{dr}{r^2}$$

$$\Delta V = -kq \left(\frac{-1}{r}\bigg|_{r_0}^{r_f}\right) = kq \left(\frac{1}{r_f} - \frac{1}{r_0}\right)$$

or

$$V_f - V_0 = kq \left(\frac{1}{r_f} - \frac{1}{r_0}\right)$$

Since this problem asks for the potential and not the potential difference, it is assuming that the reference voltage is $V = 0$ when $r = \infty$. Applying this condition at our starting location, we have

$$V_f - 0 = kq \left(\frac{1}{r_f} - \frac{1}{\infty}\right)$$

or

$$V = \frac{kq}{r} \qquad (Eq.1)$$

You were given this result for a point charge, which is what the Van de Graaff looks like electrically if you are beyond the surface of the charged sphere.
The diameter of the sphere is given, but we need the radius, $r = 1.0\ m$. The voltage above the surface is then

$$V = \frac{(8.99 \times 10^{9}\ Nm^2/C^2)(5.00 \times 10^{-3}\ C)}{1.0\ m} = 4.50 \times 10^{7}\ V = 45\ MV$$

b) Where is the 1.00 MV location?

Take equation 1) and solve for r.

$$r = \frac{kq}{V}$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

$$r = \frac{(8.99 \times 10^9\ Nm^2/C^2)(5.00 \times 10^{-3}\ C)}{1.00 \times 10^6\ V} = 45\ m$$

c) What is the energy of a triply ionized oxygen atom if it moves from the surface of the Van de Graaff to the location derived in part b)?

The voltage difference between the two locations is

$$\Delta V = V_f - V_0 = (1.00 \times 10^6\ V) - (4.50 \times 10^7\ V) = -4.40 \times 10^7\ V$$

Since the oxygen has a positive charge of +3 we can calculate the energy two ways. The first is to multiply the potential change by 3X the fundamental charge. That will give us an answer in joules. We would then need to apply a conversion factor to get it into electron-volts. That conversion factor is the value of the fundamental charge. Therefore, they cancel each other. As a result, all we really need to do is multiply the potential difference by 3.

$$\Delta U = q\Delta V = (+3e)(-4.40 \times 10^7\ V) = -1.32 \times 10^7\ eV$$

The loss in potential energy is a gain in kinetic energy. Therefore, the oxygen ion has a kinetic energy of

$$K = 1.32 \times 10^7\ eV = 132\ MeV$$