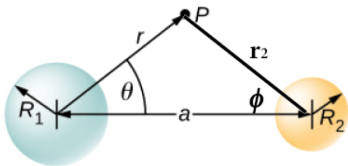
Chapter 6   Problem 86 †



Solution

Find the electric field at point P.

Since the point charges are spheres and the point of interest is beyond the surface of the spheres, we know the electric field will be the same as that for a point charge given by Coulomb's law.

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Since the magnitude of the charges are not given, we must express them in terms of the charge density and the radius of the charge.

$$Q = \tfrac{4}{3}\pi r^3 \rho$$

Let's now focus on each charge separately and find the electric field vector at point P.

The magnitude of the electric field for charge 1 is

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{4/3\,\pi R_1^3\rho_1}{r^2}\right) = \frac{R_1^3\rho_1}{3\epsilon_0 r^2}$$

Orient the x-axis as the connecting line between the two charges. The magnitude can now be broken into an x and y component.

$$\vec{E}_1 = \left(\frac{R_1^3\rho_1}{3\epsilon_0 r^2}\right)(\cos\theta\hat{i} + \sin\theta\hat{j}) \qquad (Eq.1)$$

Now to the hard part, calculating the magnitude and vector components due to charge 2.

First, we need to find the distance between charge 2 and the point P. Using the law of cosines, we get this distance. (**Notice:** If the angle θ were 90°, then the distance would be the hypotenus, but it isn't necessaryly so.)

$$r_2^2 = r^2 + a^2 - 2ar\cos\theta \qquad (Eq.2)$$

The magnitude of the electric field is then

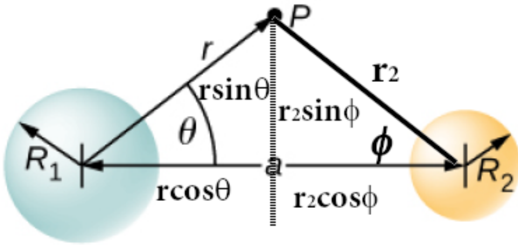
$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{4/3\,\pi R_2^3\rho_2}{r^2 + a^2 - 2ar\cos\theta}\right) = \frac{R_2^3\rho_2}{3\epsilon_0(r^2 + a^2 - 2ar\cos\theta)}$$

Now we need to break this magnitude into x and y components by introducing a second angle ϕ. Since the electric field for the second charge is directed towards the II quadrant in the proximity of point P, the electric field vector for the second charge is

$$\vec{E}_2 = \left(\frac{R_2^3\rho_2}{3\epsilon_0(r^2 + a^2 - 2ar\cos\theta)}\right)(-\cos\phi\hat{i} + \sin\phi\hat{j}) \qquad (Eq.3)$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

We have introduced two variables, ϕ and r_2. It would be useful to get rid of them. Consider the following diagram.



Notice that

$$a = r\cos\theta + r_2\cos\phi$$

Therefore,

$$\cos\phi = \frac{a - r\cos\theta}{r_2}$$

Also

$$r\sin\theta = r_2\sin\phi$$

and

$$\sin\phi = \frac{r\sin\theta}{r_2}$$

Substituting these relationships into Eq. 3 gives

$$\vec{E}_2 = \left(\frac{R_2^3\rho_2}{3\epsilon_0(r^2+a^2-2ar\cos\theta)}\right)\left(-\frac{a-r\cos\theta}{r_2}\hat{i} + \frac{r\sin\theta}{r_2}\hat{j}\right)$$

Finally, let's get rid of r_2 using Eq. 2.

$$r_2 = (r^2+a^2-2ar\cos\theta)^{1/2}$$

We now have

$$\vec{E}_2 = \left(\frac{R_2^3\rho_2}{3\epsilon_0(r^2+a^2-2ar\cos\theta)}\right)\left(-\frac{a-r\cos\theta}{(r^2+a^2-2ar\cos\theta)^{1/2}}\hat{i} + \frac{r\sin\theta}{(r^2+a^2-2ar\cos\theta)^{1/2}}\hat{j}\right)$$

Simplifying with algebra gives

$$\vec{E}_2 = \left(\frac{R_2^3\rho_2}{3\epsilon_0(r^2+a^2-2ar\cos\theta)^{3/2}}\right)\left((-a+r\cos\theta)\hat{i} + r\sin\theta\hat{j}\right)$$

Now combine this electric field with that of charge 1, given in Eq. 1 and we have

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{R_1^3\rho_1}{3\epsilon_0 r^2}\right)(\cos\theta\hat{i}+\sin\theta\hat{j}) + \left(\frac{R_2^3\rho_2}{3\epsilon_0(r^2+a^2-2ar\cos\theta)^{3/2}}\right)\left((-a+r\cos\theta)\hat{i} + r\sin\theta\hat{j}\right)$$

$$\vec{E} = \left(\frac{R_1^3\rho_1\cos\theta}{r^2} + \frac{R_2^3\rho_2(-a+r\cos\theta)}{(r^2+a^2-2ar\cos\theta)^{3/2}}\right)\frac{\hat{i}}{3\epsilon_0} + \left(\frac{R_1^3\rho_1\sin\theta}{r^2} + \frac{R_2^3\rho_2 r\sin\theta}{(r^2+a^2-2ar\cos\theta)^{3/2}}\right)\frac{\hat{j}}{3\epsilon_0}$$