Chapter 6 Problem 58[†]

Given q = 92e $R = 7.4 \times 10^{-15} m$ $e = 1.60 \times 10^{-19} C$ $\epsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$

Solution

a) Find the electric field on an electron at a distance of $r = 3.0 \times 10^{-15} m$ from the center of a uranium atom.

Since the electron is within the nucleus of the atom, where the positive charge is uniformly distributed, we can use the forumla derived from Gauss's Law. (See Example 6.6 in the chapter.)

$$E = \frac{\rho r}{3\epsilon_0}$$

To find the charge density, we need to find the total charge in the nucleus and divide it by the volume of the nucleus. The total charge is 92 multiplied by the value of the fundamental charge.

$$q = 92e^{-} = 92(1.60 \times 10^{-19} C) = 1.47 \times 10^{-17} C$$

The volume is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (7.4 \times 10^{-15} \ m)^3 = 1.70 \times 10^{-42} \ m^3$$

The charge density is then

$$\rho = \frac{q}{V} = \frac{1.47 \times 10^{-17} C}{1.70 \times 10^{-42} m^3} = 8.66 \times 10^{24} C/m^3$$

At $r = 3.0 \times 10^{-15} m$, the electric field is then

$$E = \frac{\rho r}{3\epsilon_0} = \frac{(8.66 \times 10^{24} \ C/m^3)(3.0 \times 10^{-15} \ m)}{3(8.85 \times 10^{-12} \ C^2/Nm^2)} = 9.79 \times 10^{20} \ N/C$$

The electric force is then

$$F = qE = (1.60 \times 10^{-19} \ C)(9.79 \times 10^{20} \ N/C) = 157 \ N$$

b) Find the acceleration of the electon.

From Newton's 2nd Law the acceleration is just the force divided by the mass of the electron giving

$$a = \frac{F}{m} = \frac{157 N}{9.11 \times 10^{-31} kg} = 1.72 \times 10^{32} m/s^2$$

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)