Chapter 6   Problem 42 †

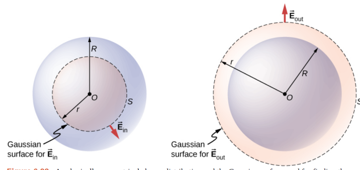

Figure 6.23 A spherically symmetrical charge distribution and the Gaussian surface used for finding the field (a) inside and (b) outside the distribution.

Given

$\rho(r) = \rho_0 r/R \qquad r \leq R$

$\rho(r) = 0 \qquad r > R$

Solution

Find the electric field both inside and outside the spherical charge distribution.

From Gauss' Law

$$\oint_S \vec{E} \cdot \vec{A} = \Phi = \frac{q_{enc}}{\epsilon_0}$$

With spherical symmetry, the area and electric field vectors are parallel, thus eliminating the dot product. Also in spherical sysmmetry, the electrical field is the same intensity anywhere on the surface. Since it is constant, the integral just adds up the surface area of a sphere. As a result, Gauss' Law with spherical symmetry is

$$E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0}$$

and the magnitude of the electric field is

$$E = \frac{q_{enc}}{4\pi\epsilon_0 r^2} \qquad (Eq.1)$$

For a radius inside the surface of the sphere, the enclosed charge is the integral of charge density throughout the volume of the sphere.

$$q_{enc} = \int \rho dV$$

Since the charge density is spherically symmetric, the integral for adding charge can use the method of shells and integrate in the radial direction.
Each shell has a surface area of a sphere and its volume is that area times dr.

$$dV = 4\pi r^2 dr$$

Inside the charge distribution, the charge density is given, so it is now a matter of performing the integral.

$$q_{enc} = \int_0^r \left(\frac{\rho_0 r}{R}\right)(4\pi r^2 dr) = \frac{4\pi\rho_0}{R}\int_0^r r^3 dr$$

$$q_{enc} = \left(\frac{4\pi\rho_0}{R}\right)\frac{r^4}{4} = \frac{\pi\rho_0 r^4}{R}$$

From equation (1) the electric field inside the distribution is then

$$E = \frac{(\pi\rho_0 r^4/R)}{4\pi\epsilon_0 r^2} = \frac{\rho_0 r^2}{4\epsilon_0 R}$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

Once you are outside the charge distribution, the charge enclosed no longer increases with radius. Once the surface, R is reached q_{enc} remains constant. This enclosed charge is

$$q_{enc} = \int_0^R \left(\frac{\rho_0 r}{R}\right)(4\pi r^2 dr) = \frac{4\pi\rho_0}{R}\int_0^R r^3 dr$$

$$q_{enc} = \left(\frac{4\pi\rho_0}{R}\right)\frac{R^4}{4} = \pi\rho_0 R^3$$

From equation (1) the electric field outside the distribution will now have a magnitude of

$$E = \frac{(\pi\rho_0 R^3)}{4\pi\epsilon_0 r^2} = \frac{\rho_0 R^3}{4\epsilon_0 r^2}$$