Chapter 6 Problem 35 †



Solution

Find the flux through the shaded face of the cube.

Imagine that the illustrated cube is just one octant of a larger cube that is 2a on a side. The charge would be at the center of this larger cube. The area of the shaded region in our picture is a^2 , but one side of the larger cube would have an area of $4a^2$. Since a cube has six identical sides, the total surface area of the larger cube is $24a^2$.

The total flux leaving the larger cube is given by Gauss's law.

$$\Phi_{total} = \frac{Q_{enclosed}}{\varepsilon_0}$$
$$\Phi_{total} = \frac{q}{\epsilon_0}$$

Since the large cube has a surface area that is 24 times larger than the shaded surface on our diagram, then the flux going through the shaded surface is

$$\Phi_{shaded} = \frac{q}{24\epsilon_0}$$

This calculuation only works because of symmetry and how we set up the larger cube. The charge was at the larger cube's center and the surface of that cube consists of 24 squares (identical to our shaded region) that are the same distance and have the same angular relationship to the charge.

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)