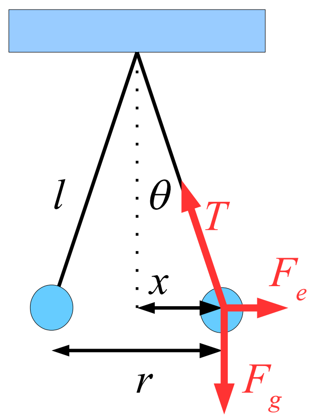
Chapter 5 Problem 68 †



Given
$k = \frac{1}{4\pi\epsilon_0}$

Solution
Find how θ depends on l and q.

First a free-body diagram needs to be generated. There are three forces: one due to electric repulsion to the right, one due to gravity in the downward direction and the last due to tension in the string to the upper-left. Using the angle designated in the diagram and let the positive x-axis be to the right and positive y-axis in the upward direction, then by Newton's 2nd law we have the following equation.

$$\Sigma\vec{F}_i = ma = \vec{F}_T + \vec{F}_e + \vec{F}_g$$

Now

$$\vec{F}_T = -T\sin\theta\hat{\imath} + T\cos\theta\hat{\jmath}$$

$$\vec{F}_e = k\frac{q_1 q_2}{r^2}\hat{\imath} = k\frac{qq}{r^2}\hat{\imath} = k\frac{q^2}{r^2}\hat{\imath}$$

$$\vec{F}_g = -mg\hat{\jmath}$$

Substitute the individual forces into Newton's 2nd law and set the acceleration equal to zero (this is a statics problem).

$$ma = 0 = -T\sin\theta\hat{\imath} + T\cos\theta\hat{\jmath} + k\frac{q^2}{r^2}\hat{\imath} - mg\hat{\jmath}$$

In the x-direction we get

$$0 = -T\sin\theta + k\frac{q^2}{r^2}$$

or

$$T\sin\theta = k\frac{q^2}{r^2}$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

In the y-direction we get

$$0 = T\cos\theta - mg$$

or

$$T\cos\theta = mg$$

Dividing the x-direction equation by the y-direction equation we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{k\frac{q^2}{r^2}}{mg}$$

Simplifying this equation gives

$$\frac{\sin\theta}{\cos\theta} = k\frac{q^2}{mgr^2}$$

$$\tan\theta = k\frac{q^2}{mgr^2} \qquad (Eq.1)$$

Now r is the distance between the two balls and $r = 2x$. x is the opposite side of a right triangle. Therefore,

$$\sin\theta = \frac{x}{l} = \frac{r/2}{l}$$

Therefore,

$$r = 2l\sin\theta$$

Substitute this into equation (1) and replace k with $1/4\pi\epsilon_0$.

$$\tan\theta = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{q^2}{mg(2l\sin\theta)^2}$$

Simplifying gives

$$\sin(\theta)^2\tan(\theta) = \frac{q^2}{(4\pi\epsilon_0)mg4l^2}$$

$$\sin(\theta)^2\tan(\theta) = \frac{q^2}{16\pi\epsilon_0 gl^2 m}$$