## Chapter 9 Problem 39 †

**Given**

Three equal masses

$\vec{r}_1 = \{(6t^2 + 5)\hat{i}\}$

$\vec{r}_2 = \{(4t + 3)\hat{i} + 4t\hat{j}\}$

$\vec{r}_3 = \{(8t)\hat{i} + (t + 4)\hat{j}\}$

**Solution**

a) Find the position of the center of mass.

Assume that each of the masses have a value of 1 $kg$. Then the total mass is

$$M = 3\ kg$$

The position of the center of mass is then

$$\vec{R} = \frac{\Sigma m_i \vec{r}_i}{M} = \frac{m \Sigma \vec{r}_i}{M}$$

$$\vec{R} = \frac{(1\ kg)\left(\{(6t^2 + 5)\hat{i}\} + \{(4t + 3)\hat{i} + 4t\hat{j}\} + \{(8t)\hat{i} + (t + 4)\hat{j}\}\right)}{3\ kg}$$

$$\vec{R} = \left\{(2t^2 + 4t + \tfrac{8}{3})\hat{i} + (\tfrac{5}{3}t + \tfrac{4}{3})\hat{j}\right\}$$

b) Find the velocity of the center of mass.

From the position of the center of mass, take the first derivative wrt. time and get the velocity.

$$\vec{V} = \frac{d\vec{R}}{dt} = \frac{d\left\{(2t^2 + 4t + \tfrac{8}{3})\hat{i} + (\tfrac{5}{3}t + \tfrac{4}{3})\hat{j}\right\}}{dt}$$

$$\vec{V} = \left\{(4t + 4)\hat{i} + (\tfrac{5}{3})\hat{j}\right\}$$

c) Find the acceleration of the center of mass.

From the velocity of the center of mass, take the first derivative wrt. time and get the acceleration.

$$\vec{A} = \frac{d\vec{V}}{dt} = \frac{d\left\{(4t + 4)\hat{i} + (\tfrac{5}{3})\hat{j}\right\}}{dt}$$

$$\vec{A} = 4\hat{i}$$

† Problem from Essential University Physics, Wolfson