

Chapter 6 Problem 54 †

Given

$$F = a\sqrt{x} = ax^{1/2}$$

$$a = 9.5 \text{ N/m}^{1/2}$$

Solution

a) Find the work done by this force from $x = 0 \text{ m}$ to $x = 3.0 \text{ m}$.

Since the force is changing with position, we use the following definition of work.

$$W = \int F dx$$

Integrate the force between x_1 and x_2 gives

$$W_{x_2-x_1} = \int_{x_1}^{x_2} ax^{1/2} dx = \frac{2}{3}ax^{3/2}\Big|_{x_1}^{x_2} = \frac{2}{3}a \left(x_2^{3/2} - x_1^{3/2} \right) \quad (1)$$

Now substitute in $x_1 = 0 \text{ m}$ and $x_2 = 3.0 \text{ m}$ and $a = 9.5 \text{ N/m}^{1/2}$

$$W = \frac{2}{3}(9.5 \text{ N/m}^{1/2}) \left((3.0 \text{ m})^{3/2} - (0 \text{ m})^{3/2} \right)$$

$$W = 32.9 \text{ J}$$

b) Find the work done by this force from 3.0 m to 6.0 m .

Equation (1) can be used with the limits of $x_1 = 3.0 \text{ m}$ and $x_2 = 6.0 \text{ m}$.

$$W = \frac{2}{3}(9.5 \text{ N/m}^{1/2}) \left((6.0 \text{ m})^{3/2} - (3.0 \text{ m})^{3/2} \right)$$

$$W = 60.2 \text{ J}$$

c) Find the work done by this force from 6.0 m to 9.0 m .

Equation (1) can be used with the limits of $x_1 = 6.0 \text{ m}$ and $x_2 = 9.0 \text{ m}$.

$$W = \frac{2}{3}(9.5 \text{ N/m}^{1/2}) \left((9.0 \text{ m})^{3/2} - (6.0 \text{ m})^{3/2} \right)$$

$$W = 77.9 \text{ J}$$

†Problem from Essential University Physics, Wolfson