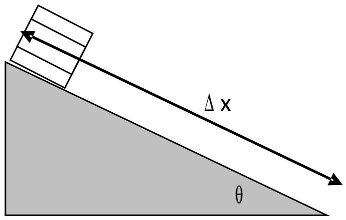
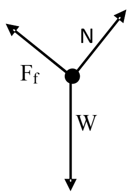
Chapter 5 Problem 58 †



Given
$\Delta x = 5.4\ m$
$\theta = 30°$
$t = 3.3\ s$
$v_0 = 0\ m/s$

Solution
Free-body diagram of the crate.



Find the maximum coefficient of friction that allows the crate to reach the bottom of the ramp in at least 3.3 s.

The slope of the ramp and the coefficient of friction are constant; therefore, the acceleration of the crate down the ramp will also be constant.
Applying the kinematic equations to the problem, find the acceleration of the crate.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Using the initial velocity of 0 m/s and an initial position of 0 m, the acceleration is

$$x = \tfrac{1}{2}at^2$$

$$a = \frac{2x}{t^2}$$

$$a = \frac{2(5.4\ m)}{(3.3\ s)^2} = 0.99\ m/s^2$$

From the free-body diagram given above and using Newton's 2^{nd} law gives

$$\Sigma\vec{F} = m\vec{a}$$

$$\vec{F}_f + \vec{W} + \vec{N} = m\vec{a}$$

Choose the coordinate system so that the x axis is down the ramp. The acceleration will be in the x direction. The weight of the crate in this coordinate system is

$$\vec{W} = mg\sin\theta\hat{i} - mg\cos\theta\hat{j}$$

†Problem from Essential University Physics, Wolfson

Writing out Newton's 2^{nd} law in unit vector notation gives

$$-\mu_k N\hat{i} + mg\sin\theta\hat{i} - mg\cos\theta\hat{j} + N\hat{j} = ma\hat{i}$$

The x-component of this equation is

$$-\mu_k N + mg\sin\theta = ma \tag{1}$$

and the y-component of this equation is

$$-mg\cos\theta + N = 0 \tag{2}$$

From equation (2) we see that the normal force is $mg\cos\theta$. Substitute this into equation(1) and solve for the acceleration.

$$-\mu_k mg\cos\theta + mg\sin\theta = ma$$

$$\mu_k = \frac{mg\sin\theta - ma}{mg\cos\theta}$$

$$\mu_k = \tan\theta - \frac{a}{g\cos\theta}$$

$$\mu_k = \tan(30°) - \frac{0.99\ m/s^2}{(9.8\ m/s^2)\cos 30°} = 0.46$$

Therefore, the coefficient of friction must be less than or equal to 0.46.