Chapter 4 Problem 62 †

(a)

(b)

Solution

a) Find the equivalent spring constant of the two springs in parallel.

The two springs are each stretched a distance of Δx. Therefore, the force exerted on the wall by each spring is

$$F_1 = -k_1 \Delta x$$

$$F_2 = -k_2 \Delta x$$

The total force exerted on the wall by the parallel springs is then

$$F_p = F_1 + F_2 = -k_1 \Delta x - k_2 \Delta x$$

$$F_p = -(k_1 + k_2)\Delta x \tag{1}$$

If this force were provided by a single spring of spring constant k_p, then

$$F_p = -k_p \Delta x \tag{2}$$

Substituting equation 1 into 2 gives

$$-(k_1 + k_2)\Delta x = -k_p \Delta x$$

Solving for k_p gives an equivalent spring constant of

$$k_p = k_1 + k_2$$

b) Find the equivalent spring constant of the two springs in series.

Now the force exerted by each spring is related to the distance they are stretched. Since the springs are placed in series, spring 1 will stretch a different distance than spring 2 because their spring constants are different. The relationship between distance and force is

$$F_1 = -k_1 \Delta p \tag{3}$$

$$F_2 = -k_2 \Delta q \tag{4}$$

$$F_s = -k_s \Delta x \tag{5}$$

The total distance stretched by the two springs has to be equal to the distance stretched by the equivalent single spring. Therefore,

$$\Delta x = \Delta p + \Delta q \tag{6}$$

†Problem from Essential University Physics, Wolfson

Using equations 3, 4, and 5 and replacing the distances involved gives

$$\frac{F_s}{-k_s} = \frac{F_1}{-k_1} + \frac{F_2}{-k_2} \tag{7}$$

Since the springs are connected in series, the forces exerted on them are the same. This is also the same as the force exerted on the single equivalent spring. Therefore,

$$F_1 = F_2 = F_3 \tag{8}$$

Substituting this into equation 7 and solving for k_s gives

$$\frac{F_s}{-k_s} = \frac{F_s}{-k_1} + \frac{F_s}{-k_2}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\frac{1}{k_s} = \frac{k_1 + k_2}{k_1 k_2}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$