

Given

$$\begin{split} m &= 630 \; Mg = 630 \times 10^6 \; g = 6.30 \times 10^5 \; kg \\ v_0 &= 0 \; m/s \\ v_f &= 7200 \; km/h \\ \Delta t &= 2.0 \; min = 120 \; s \end{split}$$

Solution

a) Find the thrust of the Ares I engine.

Find convert the final velocity into m/s.

$$v_f = \frac{7200 \ km}{hr} \left(\frac{1 \ hr}{3600 \ s}\right) \left(\frac{1000 \ m}{1 \ km}\right) = 2000 \ m/s$$

Next calculate the average acceleration of the rocket

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{2000 \ m/s - 0 \ m/s}{120 \ s} = 16.67 \ m/s^2$$

The two forces acting on the rocket are the thrust and the weight. From Newton's 2nd law

$$\Sigma \vec{F} = \vec{F}_{thrust} + \vec{W} = m\vec{a}$$

Solving for thrust gives

$$\vec{F}_{thrust} = m\vec{a} - \vec{W}$$

The weight vector is

$$\vec{W} = -mg\hat{j}$$

and the acceleration upward at 16.67 m/s^2 is represented as the vector

$$\vec{a} = 16.67 \ m/s^2 \hat{j}$$

This makes the force of thrust

$$\vec{F}_{thrust} = m\vec{a} - \vec{W} = ma\hat{j} - (-mg\hat{j})$$

$$\vec{F}_{thrust} = ma\hat{j} + mg\hat{j} = m(a+g)\hat{j}$$

$$\vec{F}_{thrust} = (6.30 \times 10^5 \ kg)(16.67 \ m/s^2 + 9.80 \ m/s^2)\hat{j}$$

$$\vec{F}_{thrust} = (6.30 \times 10^5 \ kg)(26.47 \ m/s^2)\hat{j} = 1.67 \times 10^7 \ \hat{j} \ N$$
(1)

b) Find the force on an astronaut with a mass of $75 \ kg$.

The astronaut is accelerating at the same rate as the shuttle, so formula (1) is still applicable except with a mass of 75 kg.

$$\vec{F}_{astronaut} = (75 \ kg)(16.67 \ m/s^2 + 9.80 \ m/s^2)\hat{j}$$
$$\vec{F}_{astronaut} = (75 \ kg)(26.47 \ m/s^2)\hat{j} = 1.98 \times 10^3 \ \hat{j} \ N$$

[†]Problem from Essential University Physics, Wolfson