Chapter 14 Problem 46 [†]

Given

Figure 14-36

Solution

Write out the mathematical description of the wave.

The generic form of the displacement of the wave is

$$y = A\cos(kx + \omega t)$$

The amplitude, A, is the maximum displacement from equilibrium. From Figure 14-36 the amplitude is $1.5 \ cm$.

The wavelength, λ , is the distance between crests of the wave. From Figure 14-36 the wavelength is 8 cm. Wavenumber, k, can now be calculated from the wavelength.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(8.0 \text{ cm})} = 0.785 \text{ cm}^{-1}$$

In 2.6 seconds the wave shifted 2 cm toward the positive x direction. The velocity of the wave is then

$$v = \frac{\Delta x}{\Delta t} = \frac{2.0 \text{ cm}}{2.6 \text{ s}} = 0.769 \text{ cm/s}$$

The relationship between velocity and angular frequency is given by

$$v = \frac{\omega}{k}$$

Therefore, the angular frequency is

$$\omega = kv = (0.785 \text{ cm}^{-1})(0.769 \text{ cm/s}) = 0.604 \text{ s}^{-1}$$

Since the wave is propagating in the positive x direction, the time dependent portion must be subtracted from the x dependent portion. Combining all this information gives a wave function of

$$y = (1.5 cm) \cos((0.785 cm^{-1})x - (0.604 s^{-1}) t)$$

[†]Problem from Essential University Physics, Wolfson