

Chapter 14 Problem 45 †

Given

$$\lambda = 656 \text{ nm} = 6.56 \times 10^{-7} \text{ m}$$

$$\lambda' = 708 \text{ nm} = 7.08 \times 10^{-7} \text{ m}$$

$$v = 3.0 \times 10^8 \text{ m/s}$$

Solution

Find the motion of the galaxy emitting the light.

The Doppler effect for a moving source is

$$f' = \frac{f}{1 \pm u/v} \tag{1}$$

Frequency is related to wavelength by the relationship

$$v = f\lambda$$

Replacing the frequency dependence with wavelength in Equation 1 gives

$$\frac{v}{\lambda'} = \frac{v}{\lambda} \frac{1}{1 \pm u/v}$$

$$\lambda' = \lambda(1 \pm u/v)$$

If the source is moving away from the observer the positive sign is used. Therefore,

$$\lambda' = \lambda(1 + u/v)$$

Solving for the velocity of the source, u , gives

$$u = v \left(\frac{\lambda'}{\lambda} - 1 \right)$$

Since we are working with light waves, the velocity, v , will be the speed of light. Substitute in the appropriate values gives

$$u = (3.0 \times 10^8 \text{ m/s}) \left(\frac{708 \text{ nm}}{656 \text{ nm}} - 1 \right) = 2.38 \times 10^7 \text{ m/s}$$

Compared with the speed of light, this is 7.9% the speed of light. The positive answer for u means that our choice of having the source move away from the observer was correct. Had it been negative the source would be moving towards the observer.

Therefore, the galaxy is moving away from us at $2.38 \times 10^7 \text{ m/s}$.

†Problem from Essential University Physics, Wolfson