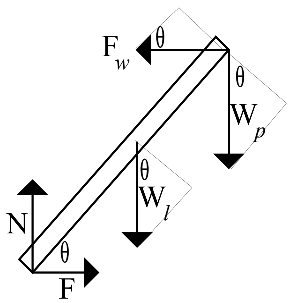
Chapter 12 Problem 41 †



Given
$l = 5.0\ m$
$m_l = 9.5\ kg$
$\theta = 66°$
$\mu = 0.42$

Solution
Find the mass of the heaviest person that can get to the top of the ladder.

The frictional force at the base of the ladder is related to the normal force by the coefficient of friction.

$$F = \mu N \tag{1}$$

From Newton's 2^{nd} law the x-component equation is

$$F - F_w = 0 \tag{2}$$

From Newton's 2^{nd} law the y-component equation is

$$N - W_l - W_p = 0 \tag{3}$$

Choosing the pivot point at the base of the ladder gives a torque equation of

$$-\tfrac{1}{2} l W_l \cos\theta - l W_p \cos\theta + l F_w \sin\theta = 0 \tag{4}$$

From equation 3 the normal force is

$$N = W_l + W_p$$

Substituting this into equation 1 gives a frictional force

$$F = \mu(W_l + W_p)$$

Substituting this into equation 2 and solving for F_w gives

$$F_w = \mu(W_l + W_p)$$

Substituting this into equation 4 gives the equation

$$-\tfrac{1}{2} l W_l \cos\theta - l W_p \cos\theta + l\mu(W_l + W_p)\sin\theta = 0$$

†Problem from Essential University Physics, Wolfson

Solving for W_p gives

$$-lW_p\cos\theta + l\mu W_p\sin\theta = \tfrac{1}{2}lW_l\cos\theta - l\mu W_l\sin\theta$$

$$W_p = \frac{\frac{1}{2}lW_l\cos\theta - l\mu W_l\sin\theta}{-l\cos\theta + l\mu\sin\theta} = \frac{W_l(\frac{1}{2}\cos\theta - \mu\sin\theta)}{-\cos\theta + \mu\sin\theta} = \frac{m_l g(\frac{1}{2}\cos\theta - \mu\sin\theta)}{-\cos\theta + \mu\sin\theta}$$

$$W_p = \frac{(9.5\ kg)(9.80\ m/s^2)(\frac{1}{2}\cos(66^\circ) - (0.42)\sin(66^\circ))}{-\cos(66^\circ) + (0.42)\sin(66^\circ)}$$

$$W_p = 729\ N$$

$$m_p = \frac{W_p}{g} = \frac{729\ N}{9.80\ m/s^2} = 74.3\ kg$$