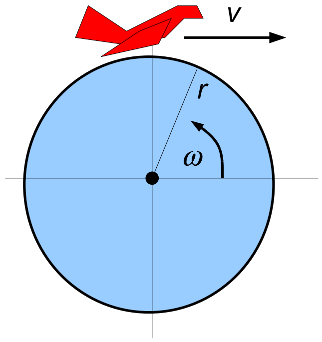
Chapter 11 Problem 47 †



Given
$r = 19\ cm = 0.19\ m$
$I_f = 0.12\ kg \cdot m^2$
$\omega_f = 5.6\ rpm$
$m_b = 140\ g = 0.140\ kg$
$v_b = 1.1\ m/s$

Solution
Find the rotation rate after the bird lands on the feeder.

First convert the angular velocity of the feeder into radians per second.

$$\omega_f = 5.6 rev/min \left(\frac{1\ min}{60\ s}\right)\left(\frac{2\pi\ rad}{1\ rev}\right) = 0.586\ rad/s$$

Just before the bird lands on the feeder, the moment of inertia of the bird is essentially that of a point mass at a distance r from the center of the feeder. Therefore,

$$I_b = m_b r^2$$

The angular velocity of the bird just before landing is negative because it is a clockwise rotation relative to the center of the feeder and has a value of

$$\omega_b = -\frac{v_b}{r}$$

The angular momentum of the bird just before landing is then

$$L_b = I_b \omega_b = -m_b r^2 \frac{v_b}{r} = -m_b r v_b$$

The angular momentum of the feeder before the bird lands is

$$L_f = I_f \omega_f$$

After the bird lands, the moment of inertia of bird/feeder combination is

$$I_T = I_b + I_f = m_b r^2 + I_f$$

and the angular momentum after the landing is

$$L_T = I_T \omega_T = (m_b r^2 + I_f)\omega_T$$

†Problem from Essential University Physics, Wolfson

Using conservation of angular momentum we have

$$L_b + L_f = L_T$$

$$-m_b r v_b + I_f \omega_f = (m_b r^2 + I_f)\omega_T$$

Solving for the angular velocity after the landing gives

$$\omega_T = \frac{-m_b r v_b + I_f \omega_f}{m_b r^2 + I_f}$$

Substitute in the provided values

$$\omega_T = \frac{-(0.140\ kg)(0.19\ m)(1.1\ m/s) + (0.12\ kg \cdot m^2)(0.586\ rad/s)}{(0.140\ kg)(0.19\ m)^2 + 0.12\ kg \cdot m^2}$$

$$\omega_T = \frac{-0.0293\ kg \cdot m^2/s + 0.0703\ kg \cdot m^2/s}{0.125\ kg \cdot m^2} = 0.328\ rad/s$$

Converting this into revolutions per minutes gives

$$\omega_T = 0.328\ rad/s \left(\frac{1\ rev}{2\pi\ rad}\right)\left(\frac{60\ s}{1\ min}\right) = 3.1\ rpm$$