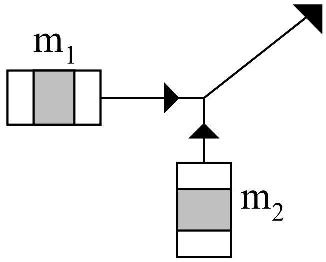
Chapter 9 Problem 63 †



Given
$m_1 = 1200\ kg$
$m_2 = 2200\ kg$
$\Delta x = 22\ m$
$\mu = 0.91$

Solution
Show that one of the cars was travelling faster than 25 km/h (6.9 m/s).

From the distance they skid after the collision, we can determine the work done by friction. The magnitude of the normal force is the same as the weight of the two cars combined assuming the intersection is flat.

$$W = -\mu N\Delta x = -\mu(m_1 + m_2)g\Delta x$$

$$W = -(0.91)(1200\ kg + 2200\ kg)(9.80\ m/s^2)(22\ m)$$

$$W = -668,000\ J$$

The work done by friction is equal to the change in the kinetic energy. Therefore, the kinetic energy just after the collision is

$$W = \Delta K = K_f - K_i$$

$$K_i = K_f - W = 0\ J - (-668,000\ J) = 668,000\ J$$

From the kinetic energy the velocity of the combined cars after the collision is

$$K_c = \tfrac{1}{2}(m_1 + m_2)v_c^2$$

$$v_c = \sqrt{\frac{2K_c}{m_1 + m_2}} = \sqrt{\frac{2(668,000\ J)}{1200\ kg + 2200\ kg}} = 19.8\ m/s$$

Now use conservation of momentum to determine whether one of the cards was speeding. From conservation of momentum we have

$$\vec{p}_i = \vec{p}_f$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_c$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_c$$

† Problem from Essential University Physics, Wolfson

Assume car 1 is travelling in the x-direction and car 2 is travelling in the y-direction, then the x-component of the momentum equation is

$$m_1 v_1 = (m_1 + m_2) v_c \cos\theta \tag{1}$$

The y-component equation is

$$m_2 v_2 = (m_1 + m_2) v_c \sin\theta \tag{2}$$

If car 1 is travelling at the limit of 6.9 m/s, then from equation (1) the angle θ must be

$$\theta = cos^{-1}\left(\frac{m_1 v_1}{(m_1 + m_2) v_c}\right) = cos^{-1}\left(\frac{(1200\ kg)(6.9\ m/s)}{(1200\ kg + 2200\ kg)(19.8\ m/s)}\right)$$

$$\theta = 82.9^\circ$$

Using this in equation (2), v_2 must be

$$v_2 = \frac{(m_1 + m_2) v_c \sin\theta}{m_2} = \frac{(1200\ kg + 2200\ kg)(19.8\ m/s)\sin 82.9^\circ}{(2200\ kg)}$$

$$v_2 = 30.4\ m/s \qquad (109\ km/h)$$

If car 2 is traveling at the limit of 6.9 m/s, then from equation (2) the angle θ must be

$$\theta = sin^{-1}\left(\frac{m_2 v_2}{(m_1 + m_2) v_c}\right) = sin^{-1}\left(\frac{(2200\ kg)(6.9\ m/s)}{(1200\ kg + 2200\ kg)(19.8\ m/s)}\right)$$

$$\theta = 13.0^\circ$$

Using this in equation (1), v_1 must be

$$v_1 = \frac{(m_1 + m_2) v_c \cos\theta}{m_1} = \frac{(1200\ kg + 2200\ kg)(19.8\ m/s)\cos 13.0^\circ}{1200\ kg}$$

$$v_2 = 54.7\ m/s \qquad (197\ km/h)$$

Either way, one of the cars has to be speeding.