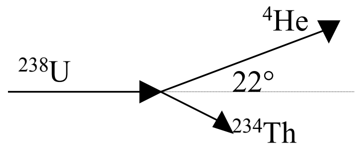
Chapter 9 Problem 48 †



Given

$m_U = 238\ AU$
$m_{Th} = 234\ AU$
$m_{He} = 4.00\ AU$
$\vec{v}_U = 5.0 \times 10^5 \hat{\imath}\ m/s$
$\vec{v}_{He} = 1.4 \times 10^7\ m/s$ 22° above the x-axis

Solution

Find the velocity of the Thorium nucleus.

First convert the Helium velocity into rectangular coordinates.

$$\vec{v}_{He} = \{v \cos\theta \hat{\imath} + v \sin\theta \hat{\jmath}\}$$

$$\vec{v}_{He} = \{(1.4 \times 10^7\ m/s)\cos(22°)\hat{\imath} + (1.4 \times 10^7\ m/s)\sin(22°)\hat{\jmath}\}$$

$$\vec{v}_{He} = \{1.30 \times 10^7 \hat{\imath} + 5.24 \times 10^6 \hat{\jmath}\}\ m/s$$

From conservation of momentum, the momentum before the nucleus decays must be equal to the momentum after the nucleus decays. Therefore,

$$\vec{p}_{before} = \vec{p}_{after}$$

$$\vec{p}_U = \vec{p}_{Th} + \vec{p}_{He}$$

$$m_U \vec{v}_U = m_{Th}\vec{v}_{Th} + m_{He}\vec{v}_{He}$$

$$m_{Th}\vec{v}_{Th} = m_U \vec{v}_U - m_{He}\vec{v}_{He}$$

$$\vec{v}_{Th} = \frac{m_U \vec{v}_U - m_{He}\vec{v}_{He}}{m_{Th}}$$

$$\vec{v}_{Th} = \frac{(238\ AU)\{5.0 \times 10^5 \hat{\imath}\ m/s\} - (4.00\ AU)\{1.3 \times 10^7 \hat{\imath}\ m/s + 5.24 \times 10^6 \hat{\jmath}\ m/s\}}{(234\ AU)}$$

$$\vec{v}_{Th} = \frac{\{1.19 \times 10^8 \hat{\imath}\} - \{5.2 \times 10^7 \hat{\imath} + 20.96 \times 10^6 \hat{\jmath}\}}{(234)}\ m/s$$

$$\vec{v}_{Th} = \frac{\{6.7 \times 10^7 \hat{\imath} - 20.96 \times 10^6 \hat{\jmath}\}}{(234)}\ m/s$$

$$\vec{v}_{Th} = \{2.86 \times 10^5 \hat{\imath} - 8.96 \times 10^4 \hat{\jmath}\}\ m/s$$

In polar coordinates the Thorium's velocity is

$$\vec{v}_{Th} = 3.0 \times 10^5\ m/s \qquad \angle -17.4°$$

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†Problem from Essential University Physics, Wolfson