Chapter 6 Problem 75 [†]

Given
$F = \frac{b}{\sqrt{x}} = bx^{-1/2}$

Solution
Show that the work done going from x_1 to x_2 is finite even in the limit when x_1 approaches zero.

Given the force function from this problem, it is clear that the force is undefined at $x = 0$. However, the definition of work is

$$W = \int F \cdot dx$$

Putting in the force function and integrating between the given limits gives

$$W = \int_{x_1}^{x_2} bx^{-1/2} dx$$

b is a constant so

$$W = b \int_{x_1}^{x_2} x^{-1/2} dx = b \left(\frac{x^{1/2}}{1/2} \right|_{x_1}^{x_2}$$

$$W = b \left(2x_2^{1/2} - 2x_1^{1/2} \right) = 2b \left(x_2^{1/2} - x_1^{1/2} \right)$$

As x_1 approaches zero, the work done approaches

$$W = 2bx_2^{1/2} = 2b\sqrt{x_2}$$

[†]Problem from Essential University Physics, Wolfson