Chapter 6 Problem 21[†]



Given k = 200 N/m

Solution

a) Find the work done to stretch the spring 10 cm.

According to the diagram, as the spring is stretched downward, the spring exerts a force in the upward direction. The relationship is as follows.

$$\vec{F_s} = -k\Delta x \hat{j}$$

By Newton's 3^{rd} law the person doing the work must be exerting an equal and opposite force on the spring. Therefore, this force is

$$\vec{F_p} = k\Delta x \hat{j}$$

We find the work that the person did on the spring by using the definition of work.

$$W = \int_0^x F dx = \int_0^x kx dx = \left. \frac{1}{2} kx^2 \right|_0^x = \left. \frac{1}{2} kx^2 \right|_0^x$$

The integral definition of work is used because the spring force is constantly changing magnitude. Therefore, to stretch the spring $10\ cm$

$$W = \frac{1}{2}(200 \ N/m)(0.10 \ m)^2 = 1.0 \ J$$

b) Find the work to go from 10 cm to 20 cm.

The work to go from $0 \ cm$ to $20 \ cm$ is

$$W = \frac{1}{2}(200 \ N/m)(0.20 \ m)^2 = 4.0 \ J$$

The difference of energy between 10 cm and 20 cm is

$$\Delta W = W_f - W_i = 4.0 \ J - 1.0 \ J = 3.0 \ J$$

$^\dagger \mathrm{Problem}$ from Essential University Physics, Wolfson