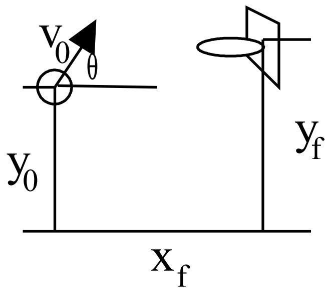
Chapter 3 Problem 67 †



Given
$y_0 = 8.2\ ft$
$y_f = 10\ ft$
$x_f = 15\ ft$
$v_0 = 26\ ft/s$
$a = -32\ ft/s^2$

Solution
Find the initial angle at which the ball is thrown.

From the initial values, the position vector is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r} = 8.2\ ft\ \hat{j} + \left\{26\ ft/s\ \cos\theta\hat{i} + 26\ ft/s\ \sin\theta\hat{j}\right\} t + \frac{1}{2}\left\{-32\ ft/s^2\ \hat{j}\right\} t^2$$

Regrouping gives

$$\vec{r} = \left\{[(26\ ft/s)t\ \cos\theta]\hat{i} + [(8.2\ ft) + (26\ ft/s)t\ \sin\theta - (16\ ft/s^2)\ t^2]\hat{j}\right\}$$

When the ball reaches the hoop, the x-component is equal to 15 ft. This gives a relationship between time and angle.

$$(26\ ft/s)t\cos\theta = 15\ ft$$

Solving for t gives

$$t = \frac{15}{26\cos\theta}\ s$$

When the ball reachees the hoop, the y-component is at 10 ft. This gives a second relationship between time and angle.

$$(8.2\ ft) + (26\ ft/s)t\sin\theta - (16\ ft/s^2)t^2 = 10\ ft$$

or

$$(16\ ft/s^2)t^2 - (26\ ft/s)t\sin\theta + 1.8\ ft = 0$$

†Problem from Essential University Physics, Wolfson

Substitute in the result from the x-component gives

$$(16\ ft/s^2)\left(\frac{15}{26\cos\theta}\ s\right)^2 - (26\ ft/s)\left(\frac{15}{26\cos\theta}\ s\right)\sin\theta + 1.8\ ft = 0$$

$$(5.33\ ft)(\cos\theta)^{-2} - (15\ ft)\tan\theta + (1.8\ ft) = 0$$

Using the definition of secant ($\sec\theta = \cos^{-1}\theta$) and the trig. identity $\sec^2\theta = 1 + \tan^2\theta$, we get

$$(5.33\ ft)(\sec\theta)^2 - (15\ ft)\tan\theta + (1.8\ ft) = 0$$

$$(5.33\ ft)(1 + \tan^2\theta) - (15\ ft)\tan\theta + (1.8\ ft) = 0$$

$$(5.33\ ft)\tan^2\theta - (15\ ft)\tan\theta + (7.13\ ft) = 0$$

Use the quadratic formula to solve for $tan\theta$.

$$\tan\theta = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(5.33)(7.13)}}{2(5.33)}$$

$$\tan\theta = 0.606\ or\ 2.21$$

Then

$$\theta = 31.2^\circ\ or\ 65.6^\circ$$

Either angle will get the ball through the hoop. The first one gets the ball there faster; however, your coach will probably want you to put more arch on the shot to get the second solution.